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Poznań – Poland
September 26–28, 2022

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All published papers received a positive opinion of the members of the Scientific Committees.

Prospective insight

The jubilee of the conference, similarly as the jubilee of every important event in our lives, invites us to reflect on the past and the future.

It was April 1960 when prof. Edmund Karaśkiewicz as a chairman of the Poznan Department of the Polish Society of Theoretical and Applied Mechanics (PTMTS) organized and headed the first two-day symposium on linear and nonlinear vibrations. It took place in Poznan. The symposium became an event organized every two years. The chairmen of the conference changed, but all of them set themselves the goal of caring for high scientific level of the symposium. It resulted in obtaining by the conference a high reputation in the Polish scientific world.

More than 60 years have passed. At that time, we observed the rapid development of technology, which fundamentally affected the world, the life of societies and every single person. The development of new technologies was possible thanks to science. On the other hand, we see how much we can support the development of science through the use of modern technical solutions. Faced with the task of organizing the 30th edition of the VIBSYS conference, we asked ourselves a number of questions. First of all, which research topics are currently the most relevant and important from the scientific and application point of view. The second issue was to define an attractive way to exchange knowledge, popularize science and encourage young scientists to conduct research.

We decided to answer the first of these questions together with the conference participants who represent various modern trends in the broadly understood subject of vibrations in physical systems. The current and subsequent editions of VIBSYS will allow us to decide which of the topics are particularly worth considering during the conference. In terms of organization, we plan to maintain new ideas that turned out to be right during the conference in 2020. These include a hybrid form of participation both stationary face-to-face on the spot and remote via an online platform, a competition for young scientists on the best presentation of the research results, popularization of history and art through trips to interesting places in the Greater Poland region and the emission of short films encouraging to see, e.g., the exhibition of the National Museum in Poznan during breaks in the sessions.

The special moment during the 30 edition of VIBSYS will be a session dedicated to the memory of prof. Czesław Cempel. Prof. Cempel worked in the Institute of Applied Mechanics and organized the VIBSYS conference many times. He was a chairman and honorary member of the scientific committee. Employees of our Institute, co-authors and friends will present the profile of the professor and his scientific achievements.

At the end of this introduction, we wish the participants fruitful discussions and many pleasant moments during the VIBSYS conference at the Poznan University of Technology.

Chairs of the Conference

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ENERGY HARVESTING USING A PIEZOELECTRIC TRANSDUCER ON EXTERNALLY FORCED BUT DAMPED OSCILLATOR

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ABSTRACT
We are focused on the investigation of the motion of a novel 3-DOF system composed of two parts. The first part contains a linear damped oscillator moving horizontally without any friction. The oscillator is connected to a piezoelectric device for the purpose of energy harvesting. The second part consists of a nonlinear damped pendulum system which is hung up at the center of the system. The dynamical model is excited by harmonic external forces. The Lagrange equations are employed to construct the governing equations, and the multiple scales technique is utilized to evaluate the analytical solutions. The analysis of the resonance scenarios and the solvability constraints yields the modulation equations. The time series of generalized coordinates of the system are analyzed. The dynamical model serves as the source of vibrations for operating the piezoelectric device in order to convert these vibrations to electrical energy. Graphical representations are used to show the effects of excitation amplitude, coupling coefficient, capacitance, load resistance, natural frequency, and damping coefficient versus the output voltage and power. The resonance shapes constructed to explore the steady-state solutions and stability analyses is carried out.

1. INTRODUCTION

Non-renewable fossil fuels are the main energy-producing resources, but they are quickly depleting and will run out within the next several decades. Energy harvesting, which captures unused ambient energy and converts it into a more useful form of energy, is the most promising renewable energy source and a perfect alternative source for energy instead of traditional sources. A piezoelectric device [1, 2] is one of the energy-harvesting devices used to transform mechanical vibrations into electrical power. In this paper, we have developed a novel physical model for energy harvesting.

2. RESULTS AND DISCUSSION

The vibrational analysis covers the system composed of the piezoelectric transducer and vibrational 3DOF mechanical system. The equations of motion are solved analytically [3], and compared with the numerical ones for more consistency and reliability (see Fig. 1). The influence of the coupling between the mechanical model and the piezoelectric device on the electrical production is represented graphically in (Fig. 2). The following non-dimensional governing equations of the model are obtained using Lagrange equations

$$\begin{aligned}
&\ddot{x} + \beta\,\ddot{z}\sin\theta + \beta\,(1+z)\ddot{\theta}\cos\theta + 2\,\beta\,\dot{z}\,\dot{\theta}\,\cos\theta - \beta\,(1+z)\dot{\theta}^2\sin\theta + c_1\dot{x} + x + \mu\,v + \omega^2 \\
&\qquad = f_1\cos p_1\tau, \\
&\ddot{z} + \ddot{x}\sin\theta - (1+z)\dot{\theta}^2 + c_2\dot{z} + \varpi^2\,z + 3\,\eta^2\alpha\,z + 3\,\eta\,\alpha\,z^2 + \alpha\,z^3 + \omega^2(1+\cos\theta) \\
&\qquad = f_2\cos p_2\tau, \\
&(1+z)^2\,\ddot{\theta} + (1+z)\,\ddot{x}\cos\theta + 2(1+z)\,\dot{z}\,\dot{\theta} + c_3\dot{\theta} + \omega^2(1+z)\sin\theta = f_3\cos p_3\tau, \\
&\dot{v} + \frac{v}{c_p\,R_L\,\omega_1} = \frac{l\,\gamma}{c_p}\dot{x}.
\end{aligned} \tag{1}$$

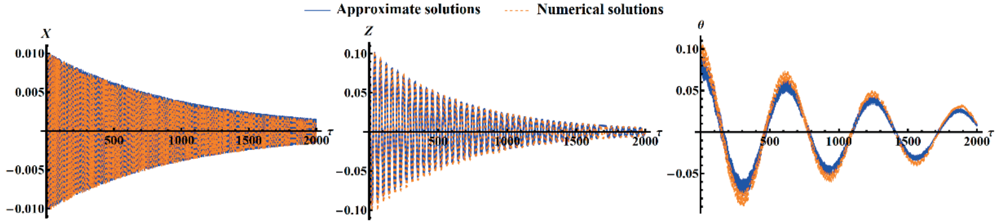

Fig. 1. Time histories of the solutions of Eqs (1).

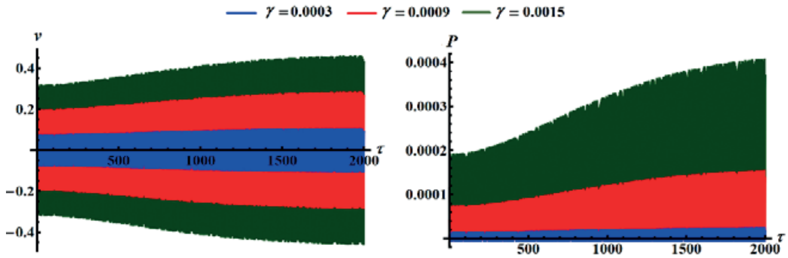

Fig. 2. The effect of different values of the coupling coefficient γ on the output voltage and power of the piezoelectric transducer.

3. CONCLUSION

A novel physical dynamical system connected with a piezoelectric harvesting transducer is investigated. Energy-harvesting technologies have a wide range of uses in daily life such as environmental monitoring, and remote medical diagnosis. The equations of motion are derived and the multiple scale technique is used to obtain the analytical solution. A comparison between the numerical and approximate solutions is represented graphically. The external resonance case is illustrated and then we get the modulation equations. The influence of the effective different parameters of the model on the output voltage and power is examined. Furthermore, resonance response curves are constructed, and then their stability has been investigated.

Acknowledgments

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STUDY OF THE CLEARANCE EFFECT IN REVOLUTE AND PRISMATIC JOINTS ON THE DYNAMICS OF A SPATIAL MECHANISM WITH FLEXIBLE LINK

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ABSTRACT

Interactions between clearance in joints and link flexibility of the spatial RPSUP mechanism (Fig. 1) are analyzed in the paper. The mathematical model of the mechanism is formulated using joint coordinates and homogeneous transformation matrices. Dynamics equations of motion are derived using the Lagrange equations of the second kind [1]. Coupler (1,3) flexibility is modeled using the Rigid Finite Element method in a modified formulation [2]. It is assumed that some joints can be imperfect, which means that dry friction and clearance effects are taken into account. The friction phenomenon is modeled using the LuGre friction model [3]. The clearance is analyzed in prismatic joint P and revolute cut-joint R. The contact force is modeled using the Lankarani-Nikravesh model [4] which is proven to be more consistent with the physics of the contact process. The closed-loop chain is transformed into two open-loop subchains with the following generalized coordinates:

$$\mathbf{q} = \left[\mathbf{q}^{(1)^T} \quad \mathbf{q}^{(2)^T} \right]^T = \left[\underbrace{\psi^{(1,1)} \quad x^{(1,2)} \quad \psi^{(1,3,0)} \quad \theta^{(1,3,0)} \quad \varphi^{(1,3,0)} \quad \mathbf{q}_f^{(1,3)^T}}_{\mathbf{q}^{(1)^T}} \;\vdots\; \underbrace{z^{(2,1)} \quad \mathbf{q}_c^{(2,1)^T} \quad \psi^{(2,2)}}_{\mathbf{q}^{(2)^T}} \right]^T \qquad (1)$$

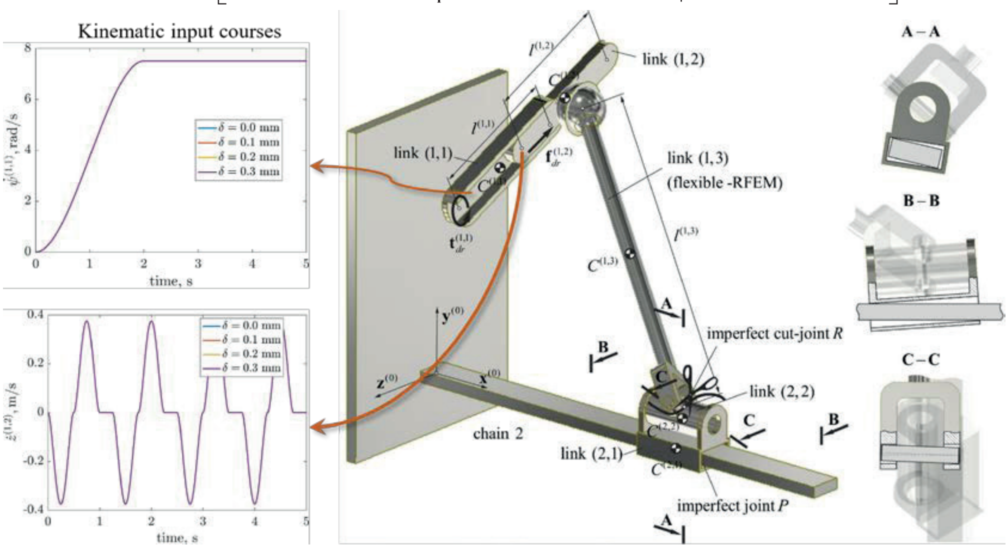

Fig. 1. The RPSUP mechanism

$\mathbf{q}_f^{(1,3)}$ - vector containing generalized coordinates of the rigid finite elements,

$\mathbf{q}_c^{(2,1)}$ - vector composed of degrees of freedom applied to describe clearance in slider (2,1).

It can be noted that different approaches should be applied to model the clearance at the cut-joint and in the other joints. If the joint is not a cut-joint then additional joint coordinates describing the clearance in the joint should be introduced. Finally, dynamics equations of motion have a form:

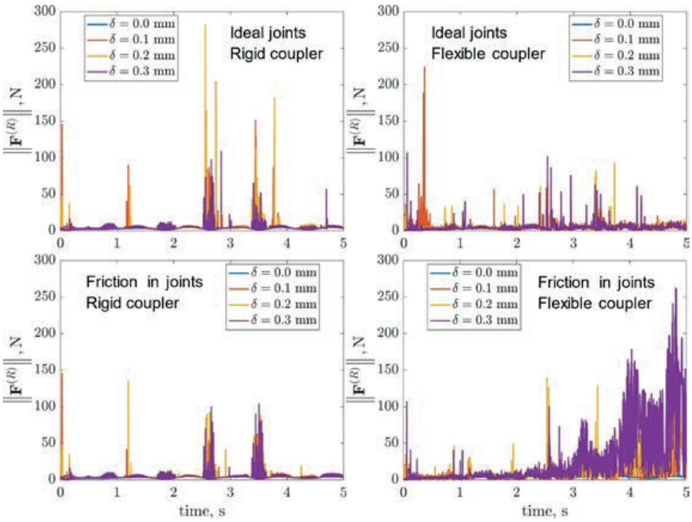

Fig. 2. Magnitude of the resultant force in cut-joint R

$$\dot{\mathbf{z}} = \mathbf{LuGre}(t, \mathbf{v}, \mathbf{z})$$

$$\begin{bmatrix} \mathbf{M}^{(1)} & \mathbf{0} & \mathbf{\Phi}_q^T \\ \mathbf{0} & \mathbf{M}^{(2)} & \mathbf{0} \\ \mathbf{\Phi}_q & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}}^{(1)} \\ \ddot{\mathbf{q}}^{(2)} \\ \mathbf{\lambda}_{dr} \end{bmatrix} = \begin{bmatrix} -\mathbf{h}^{(1)} - \mathbf{g}^{(1)} + \mathbf{Q}_{cl,r}^{(1)} + \mathbf{Q}_{flex}^{(1)} \\ -\mathbf{h}^{(2)} - \mathbf{g}^{(2)} + \mathbf{Q}_{cl,r}^{(2)} + \mathbf{Q}_{cl,p}^{(2)} \\ -\mathbf{\Gamma}(t) \end{bmatrix} \tag{2}$$

where: $\mathbf{M}^{(c)}$ - the mass matrix, $\mathbf{h}^{(c)}$ - the vector of centrifugal, Coriolis, and gyroscopic forces, $\mathbf{Q}_{cl,r}^{(c)}$, $\mathbf{Q}_{cl,p}^{(c)}$ - the generalized forces vectors resulting from clearance and friction in revolute and prismatic joints, respectively, $\mathbf{Q}_{flex}^{(1)}$ - the vector of generalized forces resulting from elastic deformations of the flexible coupler, $\mathbf{\Phi}_q$ - the constraint matrix defined for the kinematic input, $\mathbf{\lambda}_{dr}$, $\mathbf{\Gamma}(t)$ - vectors of the Lagrange multipliers and the right-hand sides of the constraint equations. Definition of all components in the dynamics equations of motion is given in [1]. In the numerical simulations, the influence of the clearance, friction in joints, and coupler flexibility on the movement of the mechanism and forces acting joints (Fig. 2) are analyzed. Some interesting results and conclusions will be presented at the conference.

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IDENTIFICATION OF SELECTED ELECTROMECHANICAL SYSTEMS USING ACQUIRED TIME-SERIES DATA

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ABSTRACT

The mathematical model of simple electromechanical systems can be derived from fundamental physical laws relating to energy and momentum. But this modelling method is subject to high bias, and it is difficult to apply when dealing with complex electromechanical systems [1]. Over the last few years, there has been an increased interest in an alternative method of modelling, which is called data-driven modelling. In this method, a machine learning tool is used to learn the dynamic behaviour of a system directly from measured data. Beside the use of time-series data, this new modelling paradigm also relies on a model structure, and a training/learning algorithm. Some common machine learning tools for data-driven modelling or identification include neural networks, sparse identification of nonlinear dynamics, and symbolic regression.

Neural networks (NNs) such as multilayer perceptron (MLP) and recurrent neural networks have been used for diverse engineering applications; their success can be linked to advancements in sensors, computational platforms, and network architectures [2]. Nevertheless, NNs have a few challenges, which include model variance with new datasets, large and quality data required, and its structure lacks physical meaning. These problems can be handled by a physics-informed neural network (PINN) [3]. The basic concept of PINN is the use of physical laws described by ordinary or partial differential equations while training a neural network to solve supervised learning problems [4]. Essentially, a PINN gives the flexibility of estimating the unknown states or variables of a system like its frictional behaviour, and it can also be used in identifying the parameters of a mathematical model.

In this abstract, we present the identification of two electromechanical systems, a geared DC motor and a double torsion pendulum system using the PINN modelling approach.

In the identification of a geared DC motor, time-series data of the system voltage input and the angular speed output were obtained experimentally. Then, a PINN model consisting of two MLP networks was proposed to predict the motor angular speed and armature current. The model loss function was formulated with the mean squared error of the model prediction and the physics-based residuals of the system. The PINN model was trained, and the network weights and biases, including the physical parameters, were updated in the process. The identified physical parameters are shown in Table 1 and the prediction results of the model are shown in Fig. 1. The results indicate the proposed PINN model can be used to accurately predict the angular speed and armature current of a geared DC motor, while also identifying the associated physical parameters of the system.

Similarly, we acquired the column and disk pendulums' time-series angular rotation data from a double torsion pendulum system. The data was used with another PINN model to predict the angular rotation of the disk pendulum and the friction between the contact surface of the pendulums. Newton's second law of rotation and the model prediction error were employed in formulating the loss of the model, and we also used the known value of each pendulum moment of inertia in the algorithm. After training the model, the predicted angular rotation of the column pendulum and the estimated planar friction are shown in Fig. 2. The overall results demonstrate the input-output relation and the frictional behaviour of an electromechanical system can be estimated using experimental time-series data from the system and a robust neural network such as PINN.

Table 1: The estimated physical parameters of a geared DC Motor

Geared DC Motor Physics Parameters							
J_{rs}	J_{os}	B_{rs}	B_{os}	K_m	L	R	K_b
0.001	0.0379	1.2539	0.3532	0.6585	0.001	1.3798	0.3938

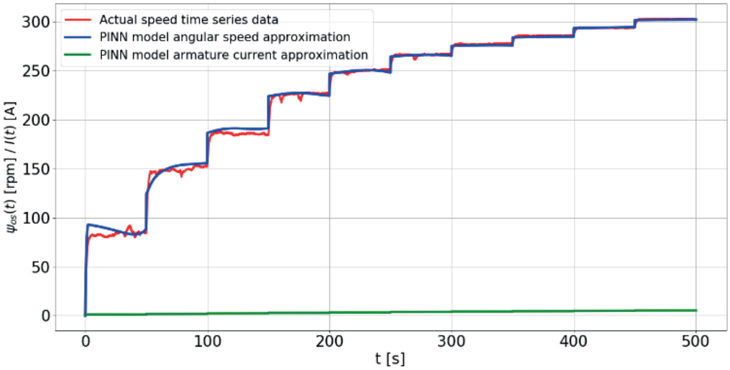

Fig. 1: The angular speed and armature current predictions of the geared DC motor PINN model after step-like increment of reference value

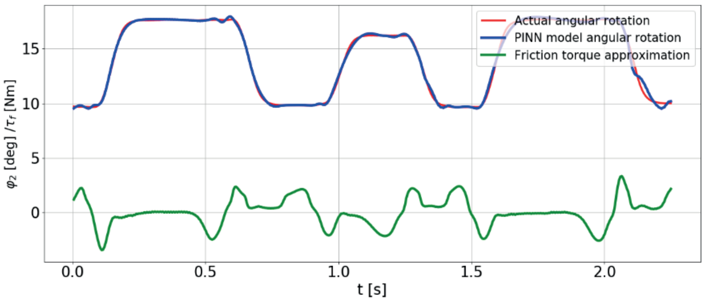

Fig. 2: The disk angular rotation and friction torque predictions of the double torsion pendulum PINN model

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NUMERICAL SIMULATION OF FUNDAMENTAL ELASTIC WAVE MODES COUPLING IN COMPOSITE PLATES

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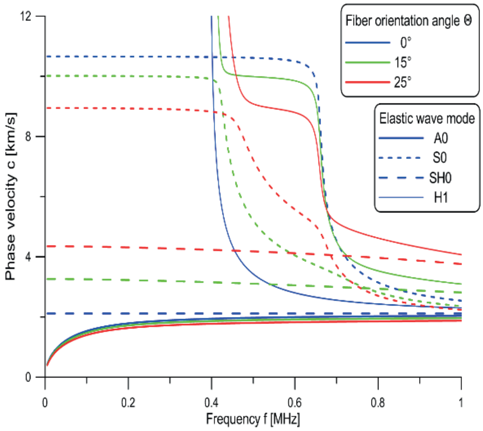
ABSTRACT

Nowadays, the phenomenon of elastic wave propagation is very often utilized in the case of different structural health monitoring (SHM) systems. However, the design of effective SHM systems demands in-depth knowledge about how the elastic waves propagate in a particular structure. Generally, there are two main problems, which significantly hinder the creation of the SHM systems. Firstly, the elastic waves propagate in the coupled form of different fundamental mods, namely symmetric (S0), shear horizontal (SH0), and anti-symmetric (A0). Moreover, each of mentioned above mods, together with the increase in excitation frequency, possesses so-called higher mods (H). Some of these wave modes are very sensitive to dispersion phenomena. In the real world, the observation and visualization of the propagation of the elastic waves are very difficult. It demands the use of advanced and very expensive equipment like, for example, laser Doppler vibrometry (LDV) [1]. There is also the possibility of the registration by, for example, piezoelectric sensors, the signals at the particular points of the structure through which the elastic waves travel.

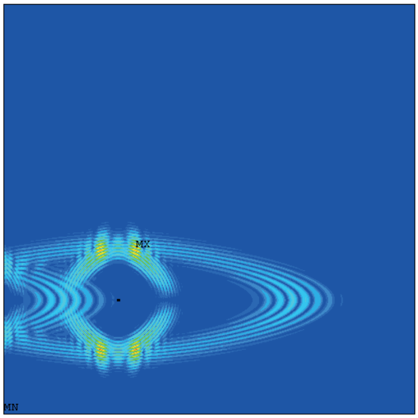
As an alternative to the experimental analysis, numerical simulations of the elastic wave propagation can be performed. The most universal approach is to use the standard finite (FE) element method [2]. The use of the FE methods makes it possible for the simulation of the elastic wave propagation in different structures of arbitrary shape and made of different materials, in particular made of composites. There is also the possibility of the selective excitation of the chosen elastic wave mode or the coupling of them.

In the current work, the propagation of the coupling of the fundamental symmetric (S0) and shear horizontal (SH0) elastic wave mods is simulated. The studied waves travel through the composite plate, which is made of carbon fiber and epoxy resin (fibers T300, matrix N5208).

At the very beginning, the so-called dispersion curves are determined. It is worth stressing that the generation of the dispersion curves, especially in the case of composite materials of arbitrary configuration should be considered a challenging task. These curves are generated with the use of our software, which is based on the stiffness matrix method [3]. The exemplary set of the dispersion curves generated for the composite plate made of carbon fiber and epoxy resin (fibers T300, matrix N5208) are shown in Fig. 1a.



a)



b)

Fig. 1. a) Dispersion curves obtained for the carbon laminate of configuration [±0°]$_4$, [±15°]$_4$ [±25°]$_4$, b) Propagation of the fundamental shear horizontal SH0 and symmetric S0 wave modes in the case of [±5°]$_4$ laminate configuration.

Next, the appropriate FE model of the plate and a piezoelectric actuator is prepared with the use of the ANSYS system, and several simulations are carried out for different composite configurations and the way how the elastic wave and/ or their coupling are excited. Fig. 1b shows the snapshot of the elastic wave mods coupling propagation in the composite plate. The coupling of the elastic waves is excited by the cubic actuator, which is perfectly bonded to the surface of the plate. As it can be observed there are two different wavefronts. The ellipsoidal wavefront depicts the propagation of the symmetric mod S0 and the circular one the shear horizontal SH0. Moreover, the elastic wave mods are identified by the comparison of their group velocity taken from the dispersion curves (Fig. 1a) and measured from the simulation of the mods. The envelope and cross-correlation [4] methods are used. The obtained results show a relatively good agreement between the group velocities obtained from dispersion curves and those, which are estimated from simulation.

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IDENTIFICATION OF ACOUSTIC PHENOMENA IN A NON-EUCLIDES METRIC SPACE

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ABSTRACT

The issue of controlling and controlling the state of acoustic threats to the environment has its own large, unrecognized scientific and application potential. It can be related to the problem of Identifying acoustic phenomena in the environment, i.e. with the requirement to build a mathematical description of the recognized phenomena, and to properly control the acoustic protection processes of the environment. It should be subject to the rules required for each identification task.

Their implementation is determined by the selection of:
• an appropriate numerical measure describing the modeled state of the recognized phenomenon,
• algebra governing the processing of the measurement results base,
• the right metric for comparing the measurement results.

An indispensable supplementary element of each identification task is the analysis of uncertainty, i.e. errors that occur in the process of conducted research diagnoses, which is an impulse to search for better, more accurate descriptions of the analysed phenomena.

With regard to the problem of identifying acoustic phenomena in the environment, there are a number of specific differences in the implementation conditions in relation to the classic identification solutions analysed in the Euclidean measurement space, which are in common use.

The article reviews them. The need to identify acoustic hazards was listed; (*present in the work environment and the human external environment*); taking into account the conditions specific to the identification processes carried out in a non-Euclidean, metric space of measurement data.

In particular, the issue of using decibel descriptions in model analysis, which, according to the psycho-physical Weber-Fechner law, are measures of the perception of acoustic pressure disorders affecting the human hearing organism, is emphasized. Their specific properties related to nonlinear formulas, processing of the measurement results database, i.e. the algebra of their modelling, are discussed. Attention was drawn to the need for a proper selection of metrics for their comparisons, corresponding to human perception of various decibel values of acoustic hazard levels. It was found that the currently used Euclidean distance measures for acoustic comparisons of measurement results in identification processes are incorrectly used in the analysis and construction of algorithms for the identification of acoustic hazards. The related application consequences are outlined. The presented proposal of new identification criteria, including measures for the comparative analysis of exceedances of permissible sound levels, free from the limitations currently operating in the control practice.

The problem of identifying acoustic phenomena in a non-Euclidean measurement space analysed in the paper was supplemented by considerations concerning the estimation of the uncertainty of the obtained

results. The issue of limitations for the process of adaptation of classical solutions of uncertainty assessment is listed; described in the "Guide to Uncertainty"; (*developed by 7 international metrology organizations*); in relation to the control tasks of the state of acoustic hazards. Possible ways of solving the problem and related methods were indicated.

The research results presented in the article and the resulting conclusions may be the basis for a wider verification of the correctness of the currently used identification procedures in non-Euclidean measurement spaces, in particular in environmental acoustic research and related decision-making processes.

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TRANSLATION OF A CABLE WITH A SUSPENDED MASS: EFFECTS ON THE VIBRATION MODES

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ABSTRACT

Ropeways have been used for century for different purposes: its advantages in terms of cost, footprint and energy efficiency make it as one of the solutions to meet the need for mobility of tomorrow. Nevertheless, because of the presence of cable which are a very flexible medium with a nonlinear geometric behavior, ropeway systems are sensitive to the effects of dynamics perturbation especially during transient phases when a braking or an acceleration occurs on the system. Unlike many other applications such as bridges or power lines, cables employed on ropeways are mobile.

So far, dynamic models for ropeways are generally made under a quasi-static assumption to account for the displacement of the mobile cables [1]. [2] is an example of study applied to ropeways which consider the cable velocity without the influence of the vehicle mass. A simple mobile cable hanging between two points was widely studied in the literature: modes calculation and parametric analysis were performed showing phenomena such as mode crossing and veering [3-4].

From this observation, the present work proposes a model of a translating cable anchored at extremities with one mass suspended. Cable dynamics is described by a general three-dimensional elastic model written in the local base attached to the cable at each point as a function of the curvilinear abscissa. Eq. (1) gives the inertial term of the cable dynamic equation with the translation speed $V_0(t)$,

$$\mu\, \boldsymbol{A}(S,t) = \mu \frac{\partial^2 \boldsymbol{X}}{\partial t^2}(S,t) + 2\mu V_0 \frac{\partial^2 \boldsymbol{X}}{\partial t\, \partial S}(S,t) + \mu {V_0}^2 \frac{\partial^2 \boldsymbol{X}}{\partial S^2}(S,t) + \mu \frac{\partial V_0}{\partial t} \frac{\partial \boldsymbol{X}}{\partial S}(S,t) \tag{1}$$

Coupled dynamic effects of both longitudinal and vertical cable movement are considered while the out-of-plane dynamics is decoupled of the in-plane movement in the linearized vision of the problem.

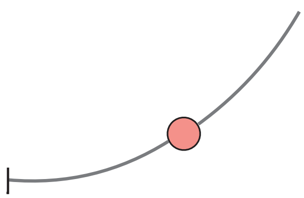

Fig. 1. Diagram of the studied model

Modal parametric analysis gives an insight into the validity area of a quasi-static assumption for the evolution according to the speed and the acceleration of the cable. The modes are defined by an analytical-numerical method considering effects of a stationary and non-stationary cable velocity coupled with the dynamics of the mass.

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STUDY OF TRANSPORT POSSIBILTIES IN THE RESONANCE ZONE OF THE NEW VIBRATORY CONVEYOR EQUIPPED WITH THE SINGLE ELECTROVIBRATOR

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ABSTRACT

Transport possibilities of the new, patented vibratory conveyor equipped with the individual vibrator, trough and additional mass were investigated in the study. In the previous work possibilities of the accurate dosing of materials by this type of devices were analysed by the authors, while in the hereby study the transport possibility at operations in the vicinity of the resonance related to the additional mass which is constituting the Frahm's eliminator [1], were revealed. It was pointed out that high transport velocities can be achieved using the vibrator of a very low unbalance due to the fact that in such situation the conveyor operates in a similar way as the resonance conveyor. Since the resonance at which the device operates is not related to the mass of the trough (on its own suspension), but to the additional mass, the conveyor is not transmitting large forces to the foundation. This is the main advantage of the proposed solution, which allows to include such conveyors in technological lines. Similar constructions placed on stiff suspensions, necessary for typical resonance conveyors, eliminates them from some technological lines [2].

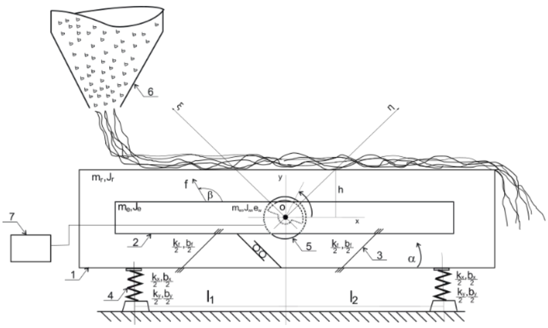

Fig. 1: Scheme of the conveyor.

The conveyor (fig. 1), according to the patent pending [3], is equipped with one unbalanced motor (5) suspended to the trough (1). Drive motor is equipped with an inverter (7) By means of the inverter the control system is able to control the rotation velocity of the motor (5). The additional mass (2) is attached to the main mass (1) on its own suspension (3). The aim of the additional mass is the introduction of the second resonance at which the conveyor will operate. When a low unbalance of the eliminator is considered it is possible to determine the trough vibration amplitudes in two directions ξ and η (fig. 2, fig. 3). It is seen, that at the frequency approaching 157 rad/s, which is on the slope of the second resonance, the amplitude enabling transport is obtained in direction ξ, while in direction η the amplitude – for the same frequency – is not causing the transport.

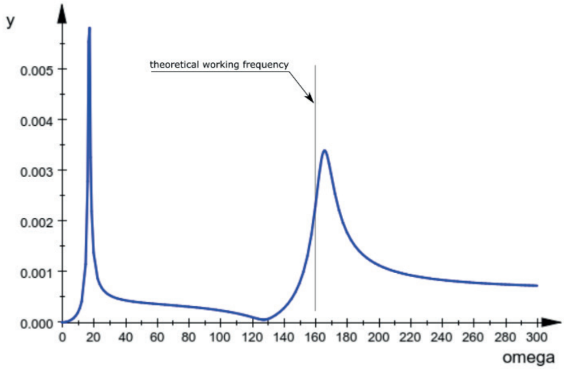

Fig. 2: Amplitude of the trough in direction of ξ.

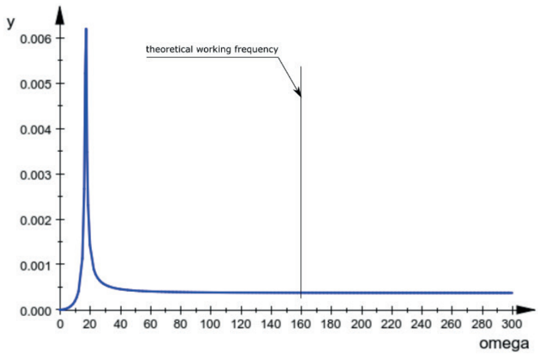

Fig. 3: Amplitude of the trough in direction of η.

The conveyor indicates very good transport properties in the resonance zone, when the drive of a low unbalance is applied. It does not transfer significant forces to the foundations, which is typical for resonance conveyors. Application of a single drive of a low unbalance provides the possibility of using a small drive motor, which means energy-saving of the conveyor.

Acknowledgments
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SUSPENSION SYSTEM WITH VARIABLE STIFFNESS FOR MOBILE ROBOTS

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ABSTRACT

The publication presents the results of work on an innovative form of the suspension node. It is a concept of semi-active suspension that allows for control the stiffness system during operation of the device. This type of suspension can be used in robotic intralogistics systems, where mobile platforms must have a suspension that ensures adequate vibration damping, keeps the platform in a horizontal position, and at the same time provides adequate pressure to the ground of the drive wheels. A particularly interesting object with the potential possibility of using such a suspension are vehicles with omnidirectional wheels. The performance of the project's suspension will be compared with the current shock absorbers on the quarter-car test bench [1].

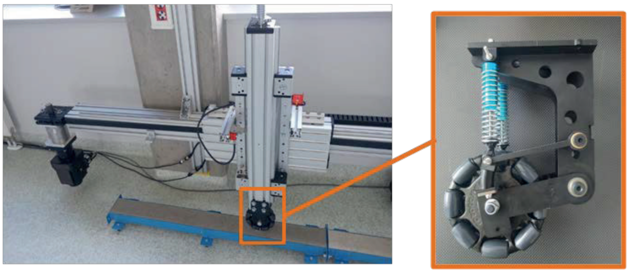

Fig. 1. Test bench for measuring vibrations in the suspension with omnidirectional wheels

The proposed design form is the first prototype version, which should primarily be used to validate computational models. The idea of the developed suspension element refers to the double torsion shafts [2] and the concept presented in [3]. A rocker arm with a wheel is attached to the developed element with variable stiffness, while the complete suspension node additionally consists of a linear shock absorber.

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VIBRATIONS OF A 3D-PRINTED FRACTIONAL CANTILEVER BEAM WITH A DOUBLE-QUICK-MOUNTS PIEZOELECTRIC TRANSDUCER

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ABSTRACT

In this work, the free vibrations of a 3D printed cantilever beam with a mass element attached to its free end and mounted double-quick-mounts piezoelectric transducer [1] are analyzed. Additionally, it is assumed that the center of mass of the element coincides with the free end point of the beam Fig. 1.

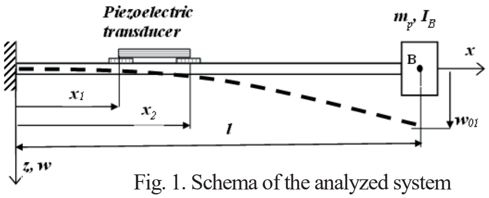

Fig. 1. Schema of the analyzed system

The analysis is carried out assuming the Euler-Bernoulli beam theory. Isotropic material properties are assumed, while the effect of printing direction is considered using different Young's modulus and mass density of the beam material. This model is formulated as

$$\sigma(t) = E\varepsilon(t) + E'_\gamma \frac{d^\gamma \varepsilon(t)}{dt^\gamma} = E\left(\varepsilon(t) + \mu_\gamma \frac{d^\gamma \varepsilon(t)}{dt^\gamma}\right) \tag{1}$$

where $\sigma(t)$ is the stress, $\varepsilon(t)$ is the strain, $\mu_\gamma = \frac{E'_\gamma}{E}$, E is the Young modulus of the beam material, E'_γ is the damping coefficient, $\frac{d^\gamma}{dt^\gamma}$ is the Caputo fractional derivative of the order γ [2]. Using the above assumptions, the equation of transverse motion of the beam can be formulated as

$$EJ = \left(\frac{\partial^4 w(x,t)}{\partial x^4} + \mu_\gamma \frac{d^\gamma}{dt^\gamma}\left(\frac{\partial^4 w(x,t)}{\partial x^4}\right)\right) + A\rho \frac{\partial^2 w(x,t)}{\partial t^2} = 0 \tag{2}$$

where $w(x,t)$ is the transversal displacement of the neutral beam axis (Fig. 1), A is the area of the cross-section of the beam, J is the moment of inertia of the beam cross-section with respect to the neutral axis, ρ is the material mass density of the beam, x is the longitudinal coordinate.

The solution to equation of motion of the beam is obtained using the mode superposition method. The solution is sought in the form

$$w(x,t) = \sum_{n=1}^{\infty} W_n(x) S_n(t) \tag{3}$$

where $S_n(t)$ are time-dependent generalized coordinates, $W_n(x)$ are the eigenfunctions of the an- alyzed beam. Using boundary conditions [3] and well-known mathematical transformations, the following equations can be derived

$$\ddot{S}_n(t) + w_n^2\left(S_n(t) + \mu_\gamma D_t^\gamma(S_n(t))\right) = 0 \tag{4}$$

where ω_n is the n-th natural undamped frequency.

The relationships for the natural frequencies, mode shapes of the beam , a method of calculating the damped natural frequencies of the analyzed beam are taken from work by Freundlich [3].

A numerical solution for free vibrations of the beam, caused be initial beam endpoint deflection is obtained by expansion of the initial conditions into eigenfunction of the beam. Assuming that the free endpoint of the beam is deflected by w_{01}, the deflection curve is as below

$$w_0(x) = \frac{w_{ol}x^2}{2l^2}\left(3 - \frac{x}{l}\right) \tag{5}$$

Using method presented in a book by Timoshenko [4], the initial condition for the beam and the element mass can be expanded in therms of time generalized coordinates and eigenfunctions

$$w_0(x) = \sum_{n=1}^{\infty} S_{0n}(t)W_n(x), w_0(l) = \sum_{n=1}^{\infty} S_{0n}(t)W_n(l), w_0'(l) = \sum_{n=1}^{\infty} S_{0n}(t){W_n}' \tag{6}$$

Multiplying both sides of the first equation (Eq. (6)) by $m_l W_m(x)$ and integrating along a beam length, next multiplying both sides of the second equation by $m_p W_m(l)$ and the third equation by $I_B W'_m(l)$, then adding these equations, and employing the orthogonality condition shown below

$$\int_0^l W_n(x)m_l W_m(x)dx + m_p \cdot W_n(l)W_m(l) + I_B \cdot W_n'(l)W_m'(l) = \delta_{nm} \tag{7}$$

the following expresion for the initial values of the time generalized coordinates is obtained

$$S_{0m}(t=0) = \frac{\int_0^l w_0(x)m_l W_m(x)dx + w_0(l)m_p W_m(l) + w_0'(l)I_B W_m'(l)}{\int_0^l m_l W_m^2(x)dx + m_p W_m^2(l) + I_B \cdot {W_m'}^2(l)} \tag{8}$$

The numerical solution to Eq. (8) is performed using a numerical scheme presented in the work by Freundlich [5].

A very simple model of a piezoelectric double-quick-monts tranduser [1] acting as a sensor is as- sumed, namely its mass and stiffness are neglected. Moreover, it is assumed that the transducer operates in bending mode (see [1]), thus the voltage generated in the transducer is proportional to the difference in deflection of its ends. Therefore, having the calculated deflections of the transducer ends, that is, for the coordinates x_1 and x_2 (see Fig. 1), the voltage generated by the transducer can be calculated. Using the equations presented above, the effect of the material and geometric parame- ters of the beam on its free vibration is examined. The obtained numerical results will be verified by experimental research.

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ANALYSIS OF VIBRO-IMPACT DYNAMICS BASED ON THE METHOD OF MULTIPLE SCALES

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ABSTRACT

Vibro-impact systems find various applications in engineering. For example, in the area of vibration suppression, passive energy absorbers in the form of vibro-impact nonlinear energy sinks (VI NES) have been extensively studied over last decades [1–4]. Although many impact oscillators have a relatively simple structure, their essential strong nonlinearity is the feature that makes it difficult or impossible to study the dynamic problems analytically. After all, many approximate analytical methods for nonlinear systems originate from the classical perturbation approach, thus they can be used only under the restriction of weak nonlinearity [6].

This paper is devoted to the approximate analytical studies based on the multiple time scales method (MTSM) combined with a use of a sawtooth wave function technique. In recent years this approach has been commonly applied to low-dimensional systems including VI NES [2–5].

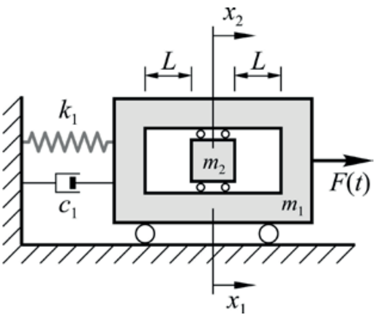

Fig. 1. Dynamical system with a standard VI NES

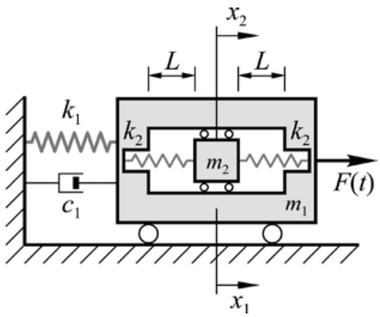

Fig. 2. Dynamical system with a hybrid VI NES

The idea of the solution strategy is presented by the example of the dynamical system with a standard VI NES (see Fig. 1). The vibro-impact unit consists of a damped linear oscillator (box) of mass m_1 and an internal block of mass m_2 moving freely between two rigid walls. The box is subjected to a harmonic force $F(t) = F_0 \sin(\omega t)$, which leads to mutual impact interactions. It is assumed that the mass of the block is relatively small ($m_2 \ll m_1$). The constants associated with the linear spring and viscous damper are denoted by k_1 and c_1, respectively.

Classically, impacts between both components of the system are characterized by the restitution coefficient κ. The duration of the collisions is assumed to be very short, and Newton's restitution rule together with the momentum conservation principle are used. As a result, the vibro-impact motion of the system (for $|x_2 - x_1| \le L$) is described by

$$\begin{aligned} m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1 x_1 &= F_0 \sin(\omega t) - \mu(1+\kappa)\sum_j (\dot{x}_1 - \dot{x}_2)\delta\big(t - t_j^-\big), \\ m_2\ddot{x}_2 &= \mu(1+\kappa)\sum_j (\dot{x}_1 - \dot{x}_2)\delta\big(t - t_j^-\big), \end{aligned} \tag{1}$$

where x_1, x_2 are the displacements of the components from equilibrium, $\mu = m_1 m_2/(m_1 + m_2)$, $\delta(\cdot)$ is the Dirac delta function, and t_j^- denotes the time instant immediately before the jth impact. The mathematical model can be presented in a non-dimensional form. Moreover, to cope with the impulsive right-hand sides of Eqs. (1), new variables are defined, and the equations are transformed to a more convenient form [5].

The analytical studies are restricted to the case of a periodic steady-state motion with two symmetric impacts per cycle near 1:1 resonance. The multiple scales method involving two time scales is used. The mass ratio $\varepsilon = m_2/m_1$ plays a role of the small parameter ($\varepsilon \ll 1$). Additionally, a sawtooth function reflecting the non-smooth nature of the solution must be employed.

Certain restrictions on the solution amplitudes are determined based on geometric constraints and stability criteria. A specific concept of the stability analysis of the periodic motion, suited for the 'MTSM – sawtooth function' technique, is presented. The whole analytical approach allows one to obtain the frequency–response and force–response curves with stable and unstable branches. Moreover, the interplay between the model parameters as well as their effect on the existence and stability of the periodic solutions are determined on various parameter planes. The graphical representation of stability boundaries is confronted with the literature results obtained with different methods [1].

Furthermore, the theoretical predictions, i.e. the amplitude of the periodic steady-state motion and the range of its stability, can be compared to the results of purely numerical simulations. To integrate the equations of motion between consecutive impacts, the Runge–Kutta–Fehlberg method (RKF45) is used. When the geometric constraints are violated, the procedure switches to the standard Runge-Kutta method (RK4) modified with the concept of time-step bisection.

On the basis of a series of numerical experiments, bifurcation diagrams and the spectra of Lyapunov exponents are presented. In the latter case, the Müller algorithm for non-smooth dynamical systems together with the Gram-Schmidt reorthonormalization are employed [7, 8].

Finally, the limitations and extensibility of the presented approximate analytical approach are discussed. In particular, an extension of the applicability of the method to the dynamic problem with the so-called hybrid VI NES (see Fig. 2) is outlined. In such a case, the internal block is attached to the box via two linear springs. Consequently, the mathematical model for the system is more complicated, and the vibro-impact solutions must be formulated with a use of a modified sawtooth function.

Acknowledgments

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INFLUENCE OF MACHINING AND DESIGN PARAMETERS OF SHAFTS ON COOPERATION OF TOOTHED GEAR

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ABSTRACT

The influence of deflection of rotating shafts on strength calculation of gears is presented in the paper. Designing machine drive systems is usually associated with the use of torque transmitting elements - the most commonly is used the combination of shafts and various types of gears. In strength tests of toothed gears, it is important to evaluate the uniformity of the distribution of the inter-tooth force along the contact line. In gear strength calculations it is evaluated by the coefficients of uniform distribution of load [1]: $K_{F\beta}$ – the face load factor for tooth-root stress, and $K_{H\beta}$ – the face load factor for contact stress. This issue is of particular importance for wide gears, i.e. when the ratio of the pinion width to the pitch diameter is greater than 1.2. The main cause of uneven pressure distribution, apart from manufacturing and assembly errors, bearing clearance and thermal deformations, are errors resulting from the deformation of gears, shafts and bodies. In most cases, a precise strength analysis taking into account the impact of the actual deformation of shafts and bodies is of significant importance for the assessment of the strength of toothed gears and thus affects their durability [2]. The importance of the problem is evidenced by the fact that the standard [3] provides various methods of determining the load distribution coefficient along the width of the rim: from method A (load distribution determined on the basis of an exhaustive analysis of all influencing factors), through B, C, to method D (method simplified).

In the paper it is taken into account the machining tolerance of cross-section of shaft journals and the estimation of its influence on the stiffness of the shaft is made. Moreover the stiffness of the shaft is calculated with taking into account the connection between shaft's journals and the wheels mounted on them – especially for the case of the press-fit connection the stiffness increases considerably. For determination the line of deflection of the shaft the finite element method is used. The non-circle cross-sections of the shaft pins are the reason of vibration appearing during rotation of shafts. These vibration causes the additional deflection of the rotating shaft and influences the gear wheel cooperation. These problem is also addressed in this paper and the vibration analysis of the shaft circulating at constant speed is also presented. The coefficients $K_{F\beta}$ and $K_{H\beta}$ are calculated for both static and dynamic cases. The analysis results indicated that even in the case of a relatively rigid shaft it is important to take into account the effects of shaft deformation on working conditions of gears. The additional calculations performed for the case when geometrical nonlinearities of the shaft are taken into account proved that their influence on coefficients of load distribution is very small and can be omitted.

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DYNAMICS OF RCM MECHANISM OF SURGICAL ROBOT FOR PERIODIC MOVEMENTS WITH CONSIDERATION OF BLDC ACTUATORS, FUZZY PID CONTROL AND GMS FRICTION MODEL

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ABSTRACT

The remote canter of motion mechanism (RCM) of a surgical robot that performs periodic movements to obtain the required shape of the operating space in the patient's chest was formulated using the block diagram method in an interactive manner in the Simulink environment.

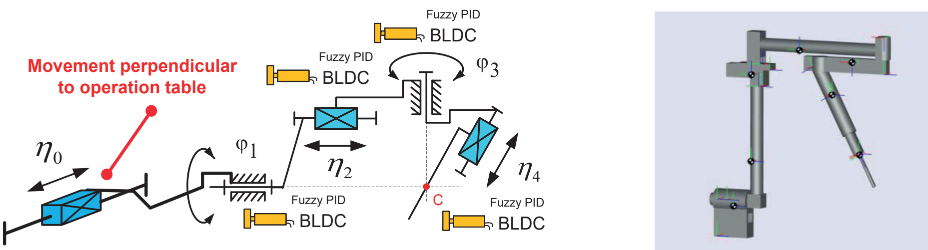

Fig.1. Novel RCM mechanism and Simulink model

The model of the dynamics of joints and links with known mechanical distributions is driven with the brushless direct current (BLDC) actuators. The correct characteristics of the control quality indicators are obtained using the fuzzy proportional – integral – derivative (PID) controller [1]. The modern Generalized Maxwell – Slip (GMS) friction model has been added to the system [2]. The dynamics model is solved by the Runge – Kutta method in the Bogacki – Sampine and Dormand – Prince variants, with ordinary differential equation (ODE) in the Matlab environment [3,4]. Force and velocity obtained during in vitro experiments on cardiovascular tissue were applied to the mechatronic system of the RCM mechanism terminated with a scalpel.

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DETERMINATION OF OPTIMAL SOLUTIONS FOR BALANCED RCM MECHANISM OF SURGICAL ROBOT DURING NATURAL VIBRATION, LINEAR BUCKLING AND SPHERICAL MOVEMENT TAKING INTO ACCOUNT INPUTS FROM IN VITRO EXPERIMENTS ON CARDIOVASCULAR TISSUE

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ABSTRACT

Remote centre of motion mechanisms (RCM) enable efficient surgical operation of the cardiovascular system with high incision accuracy, which allows for achieving excellent results in operating room. A CAD model of the innovative construction of (RCM) mechanism was created and the balances were calculated minimizing the value of the required drive torques necessary to obtain the functionality of the surgical robot. The obtained mechanical system was optimized with the following criteria: the first natural frequency, the buckling coefficient, and the von Mises reduced stress. The best solution was found from the point of view of mechanics for the adopted criteria and constraints. The boundary conditions for the model were determined on the basis of two professional in vitro experiments on cardiovascular tissue.

The values of the forces that appear on the endoscopic tool for tissue incision (scalpel, puncture needle) and velocity inputs were determined, which were selected by a medical expert for individual joints of the (RCM) mechanism. The force values were determined using a dynamometric sensor by repeatedly puncturing and cutting the tissue. The normality of the distribution of force measurement data was confirmed by the Kolmogorov – Smirnov test. The experiment was carried out at the Biocybernetics Laboratory of Zbigniew Religa Foundation of the Cardiac Surgery Development in an ultra-modern method using the Robin Heart Vision surgical robot which is illustrated in the Fig. 1. This robot directed the movement of the endoscopic camera. Discrete trajectory measurement data was filtered from random components with the use of the Savitzky – Golay (SG) polynomial filter, the parameters of which were optimally selected on the basis of the Durbin–Watson (DW) statistical test. In this method, undesirable gradients in velocity, acceleration and jerks are eliminated. Data were acquired at a frequency of 100 [Hz]. The mathematical model of multi-criteria optimization is based on the discrete finite element method (FEM) and a genetic algorithm that solves a given problem on the basis of Pareto fronts [1].

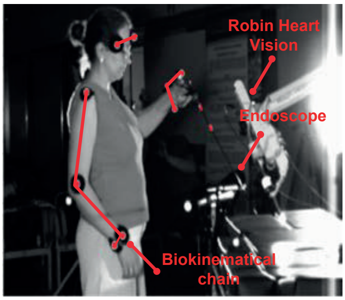

Fig. 1 In vitro cardiac surgery

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FSI SIMULATION OF FLOATING WIND TURBINE BASED ON SPH METHOD

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ABSTRACT

Simulation of floating wind turbines might be a complex issue. There are several phenomena which should be taken into consideration like Fluid Dynamics, Multibody Dynamics, Aerodynamic Forces, Aeroelastic of the blades, and FSI (Fluid-Structure interaction). There are several softwares available which can handle this task like NREL FAST [2], HAWC2 (DTU) or more sophisticated CFD-based platforms developed by scientists [1][2][4][5][7]. However, Navier-Stokes multiphase CFD simulations are time and memory consuming [1-3], therefore a new fast approach has been proposed. SPH (Smoothed Particle Hydrodynamics) is a method that is very good for FSI [6], and since it handles parallelization very well and requires low memory, it can be used on modern graphics cards. Simulation utilized on GPU could reduce computation by several times compared to Navier-Stokes CFD on CPU. As it is an early stage of the research, the blade aeroelastic will be excluded from the simulation. According to [4] the influence of this phenomena seems to be low.

For the main governing solver an open source solver DualSPHysics has been chosen. This platform calculates the body movement according to inertia and aerodynamic forces which are received from two external solvers (respectively: Chrono and FAST). Data exchange system is presented in Fig. 1.

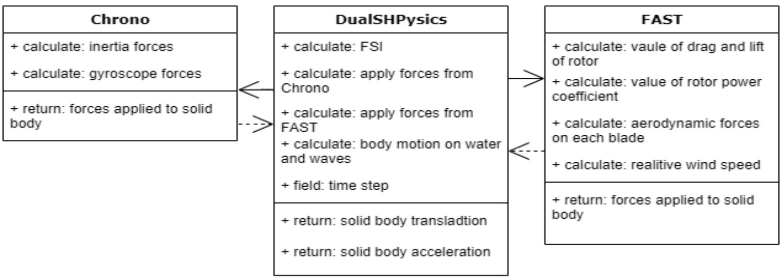

Fig. 1. Caption of the figure

Acknowledgments
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MODEL OF THE DYNAMICS OF MOTION OF A FOUR-WHEELED MOBILE PLATFORM WITH THE DYNAMIC INTERACTIONS OF DRIVE WHEEL SYSTEMS

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ABSTRACT
The model of the motion of a four-wheel mobile platform, taking into account the dynamic interactions of drive wheel systems, including the interaction of wheels with the ground, was described in the paper. The proposed computational model is a modification and extension of the model described in [1-3]. The platform dynamics model was developed in relation to the system presented schematically in Fig. 1. Modification of the model described in the above-mentioned works includes the introduction of dynamic models of drive wheels interaction with the platform to the description of the dynamics of platform motion. These models include vibrations of deformable drive wheel systems and their interaction with the ground..

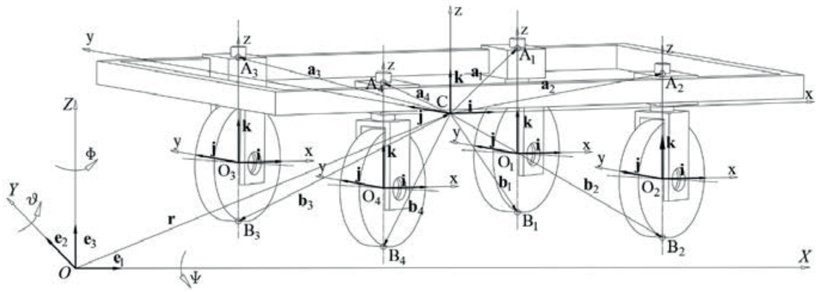

Fig. 1. Model of four-wheeled mobile platform.

On the basis of the adopted mathematical description, an algorithm was proposed and a computing program was developed in the Matlab environment for the analysis of platform motion parameters in relation to cases with set parameters of drive wheel systems as well as platform design parameters and friction parameters between the drive wheels and the ground. The work contains sample results of simulation of motion and conclusions.

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VIBRATIONS OF AXIALLY FUNCTIONALLY GRADED BEAMS WITH AXIAL FORCE

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ABSTRACT

In this contribution vibrations of functionally graded microstructured beams with axial forces are considered. The effect of microstructure is taken into account using the tolerance modelling method to model this problem. It is known that the tolerance averaging technique replaces the equation with non-continuous, tolerance-periodic, highly oscillating coefficients by the differential equations with slowly-varying coefficients. The averaged equations describe also the effect of microstructure. As an example vibrations of a simply supported functionally graded microstructured beam are shown.

Composite structures made of materials with functional gradation of properties, cf. [1], e.g. slender functionally graded microstructured beams, cf. Figure 1, are used in various engineering applications as building, aeroplane or marine engineering. Such structures are applied to obtain optimal mechanical properties.

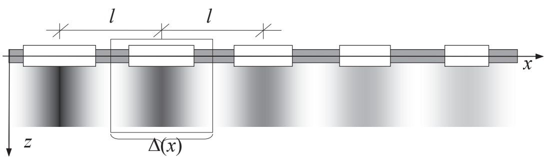

Fig. 1. Axially functionally graded beam

The size of the microstructure of considered slender axially functionally graded beams is determined by the length l of the cell □(x). Denoting: a deflection of the beam (tolerance-periodic in x) by w; an axial force by n; a load normal to the beam axis by p; a beam stiffness (tolerance-periodic in x) by EJ; the mass density □ (tolerance-periodic in x), this problem can be described by the governing equation with highly-oscillating, tolerance-periodic, non-continuous functional coefficients, i.e.:

$$\partial^2(EJ\partial^2 w) - \partial(n\partial w) + \mu\ddot{w} = p. \tag{1}$$

Different problems of functionally graded structures are analysed using the known averaging methods, proposed for periodic media, cf. [1]. Functionally graded microstructured beams can be also considered applying similar methods. Between them models were proposed, e.g.: using differential quadrature element method [2], to analyse flexural wave bandgaps in periodic plates; based on broken line hypothesis [3, 4], to consider stability of sandwich beams with microheterogeneous core; an analytical-numerical model based on analytical relations and finite element method [5], to analyse a torsion of auxetic composite beams. Unfortunately, the effect of the microstructure size is usually neglected in governing equations of the most of these averaging methods. But, *the tolerance modelling* allows to describe this effect, replacing governing equations with tolerance-periodic/periodic, non-continuous coefficients by equations with slowly-varying/constant coefficients. This method was applied for different problems of microstructured media in many papers, e.g. for vibrations or stability of periodic beams, cf. [6, 7].

The tolerance modelling procedure is based on some introductory concepts, as: a tolerance-periodic function, a slowly-varying function or an averaging operation; and on modelling assumptions. The main modelling assumption is *the micro-macro decomposition*, in which the beam deflection $w(x)$ is decomposed in the form: $w(x)=W(x,t)+g^A(x)Q^A(x,t)$, with: $g^A(\cdot)$ being the known fluctuation shape functions (tolerance-periodic in x); W and Q^A being macrodeflection and fluctuation variables, respectively (slowly-varying in x). Applying the tolerance modelling procedure to Eq. (1) and denoting:

$$\begin{aligned}
&D \equiv <EJ>, \quad D^A \equiv <EJ\partial\partial g^A>, \quad D^{AB} \equiv <EJ\partial\partial g^B \partial\partial g^A>,\\
&\tilde{m} \equiv <\mu>, \quad \tilde{m}^{AB} \equiv l^{-4} <\mu g^A g^B>,\\
&l^2 H^{AB} \equiv <\partial\partial g^A g^B>,\\
&P \equiv <p>, \quad l^2 P^A \equiv <p g^A>,
\end{aligned} \tag{2}$$

the tolerance model equations can be written as:

$$\begin{aligned}
&D\partial\partial\partial\partial W + D^A \partial\partial V^A - \partial(N\partial W) + \tilde{m}\ddot{W} = P,\\
&D^{AB} V^B + D^A \partial\partial W - l^2 H^{AB} N V^B + l^4 m^{AB} \ddot{Q}^B = l^2 P^A,
\end{aligned} \tag{3}$$

where N is the averaged axial force. Eqs. (3) are the system of N+1 differential equations with averaged slowly-varying coefficients, in contrary to Eq. (1). Because some of these coefficients involve the length of the periodicity cell, Eqs. (3) describe the vibrations of the axially functionally graded beams under axial forces, with the effect of the microstructure size. Some numerical examples will be presented in forthcoming papers.

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BIOMECHANICAL ANALYSIS OF THE JUMP SHOT IN BASKETBALL

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ABSTRACT

The jump shot is the basic activity of basketball, thanks to it players get the most points [1]. It is performed very often, which means that its incorrect technique can lead to injury. Most ankle injuries are the result of an incorrect jump landing [2]. Following work describes a study conducted on female basketball players aged 19 to 23 years old, playing basketball for a minimum of 3 years. The BTS SMART motion analysis system and dynamometric platforms were used to measure key parameters of the jump. The purpose of the study was to analyze the jump throw from the biomechanics point of view. Various parameters have been developed to assess the quality of the jump, the asymmetry of body position, and power of impact with the ground. This will allow to determine if there is a relationship between landing method and ankle injury, soft landing technique skill and experience, previous ankle injuries and landing method, asymmetry of body position in space and impact ratio. Conclusions and recommendations for further continuation of the study will also be presented.

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THE LOWER FREE VIBRATIONS FREQUENCIES OF THIN PLATES WITH FUNCTIONALLY GRADED STRUCTURE

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ABSTRACT

The paper presents free vibrations of thin tolerance-periodic plate with the functionally graded structure on the macrolevel. The region of undeformed plate is denoted by $\Omega\equiv\{(\mathbf{x}, z): -d/2\leq z\leq d/2, \mathbf{x} \in \Pi\}$, where Π is the plate midplane, $d(\cdot)$ is the plate thickness and it is assumed to be constant. The size of microlevel is characterized by $l\equiv[(l_1)^2+(l_2)^2]^{1/2}$ – length of the "basic cell". The cell is very small element compared to the plate. A fragment of the plate is shown in Fig. 1.

Dynamic problems of these plate are described by partial differential equations of fourth order with highly oscillating, non-continuous and tolerance-periodic coefficients. In order to solve these equations the tolerance averaging technique, developed by Woźniak [1, 2], is used.

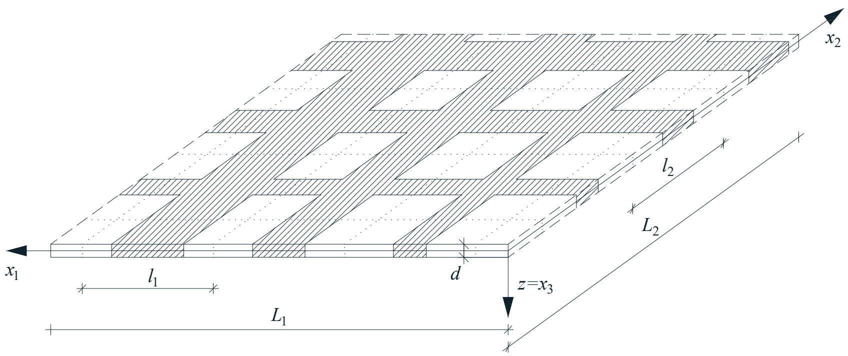

Fig. 1. A fragment of thin functionally graded plate

Following the books by [1,2] the fundamental modelling assumptions can be formulated. The first is micro-macro decomposition the plate deflection w, and the second is the tolerance averaging approximation, where the terms of an order of $0(\xi)$ are treated as negligibly small in the course of modelling. The plate deflection w has been taken in the form

$$w(\mathbf{x},t) = W(\mathbf{x},t) + h^A(\mathbf{x})V^A(\mathbf{x},t), \;\; A = 1,\dots,N, \quad \mathbf{x} \in \Pi, \tag{1}$$

where $W(\cdot,t)$, $V^A(\cdot,t)$ are the slowly-varying functions in $\mathbf{x}=(x_1,x_2)$ called the macrodeflection and the fluctuation amplitudes respectively, $h^A(\mathbf{x})$ are the known fluctuation shape functions.

Using modelling assumptions and the tolerance averaging technique the equations of functionally graded plates can be written in form

$$\begin{gathered}\partial_{\alpha\beta}(< B_{\alpha\beta\gamma\delta} > \partial_{\gamma\delta}W + < B_{\alpha\beta\gamma\delta}\partial_{\gamma\delta}h^B > V^B + < \mu > \ddot{W} - < \vartheta > \partial_{\alpha\beta}\ddot{W} = 0,\\ < B_{\alpha\beta\gamma\delta}\,\partial_{\gamma\delta}h^A > \partial_{\gamma\delta}W + < B_{\alpha\beta\gamma\delta}\,\partial_{\alpha\beta}h^A\partial_{\gamma\delta}h^B > V^B + < \mu h^A h^B > \ddot{V^B} +\\ < \vartheta\partial_\alpha h^A\partial_\beta h^B > \ddot{V^B} = 0.\end{gathered} \tag{2}$$

The equations (2) with decomposition (1) described the tolerance model for thin plates with functionally graded structure.
For the asymptotic model equations takes a form

$$\partial_{\alpha\beta}((< B_{\alpha\beta\gamma\delta} > - < B_{\alpha\beta\gamma\delta}\partial_{\gamma\delta}h^B >< B_{\alpha\beta\gamma\delta}\,\partial_{\alpha\beta}h^A\partial_{\gamma\delta}h^B >^{-1}< B_{\alpha\beta\gamma\delta}\,\partial_{\gamma\delta}h^A >)\partial_{\gamma\delta}W + \\ < \mu > \ddot{W} - < \vartheta > \partial_{\alpha\beta}\ddot{W} = 0, \tag{3}$$

$$V^B = - < B_{\alpha\beta\gamma\delta}\,\partial_{\gamma\delta}h^B >< B_{\alpha\beta\gamma\delta}\,\partial_{\alpha\beta}h^A\partial_{\gamma\delta}h^B >^{-1}\partial_{\alpha\beta}W.$$

Equations (2) and (3) have slowly-varying coefficients. The analytical solutions to describe the free vibrations of this kind of plates is very difficult to find. Therefore the known Ritz method can be used to derive approximate formulas for free vibrations frequencies. In order to obtain these formulas, the relationship of the maximal strain energy and the maximal kinetic energy is necessary to determinate. Using the conditions of the Ritz method and after some manipulations, the following formulas are obtained

$$(\omega_-^*)^2 = \frac{\breve{B}^*(\bar{\mu}^* + \bar{\vartheta}^*) + \hat{B}^*(\breve{\mu}^* + \breve{\vartheta}^*)}{2(\bar{\mu}^* + \bar{\vartheta}^*)(\breve{\mu}^* + \breve{\vartheta}^*)} + \frac{\sqrt{[\breve{B}^*(\bar{\mu}^* + \bar{\vartheta}^*) - \hat{B}^*(\breve{\mu}^* + \breve{\vartheta}^*]^2 + 4{\bar{B}^*}^2(\bar{\mu}^* + \bar{\vartheta}^*)(\breve{\mu}^* + \breve{\vartheta}^*)}}{2(\bar{\mu}^* + \bar{\vartheta}^*)(\breve{\mu}^* + \breve{\vartheta}^*)}. \tag{4}$$

for the lower ω_-^* free vibrations frequency of the tolerance model.
For asymptotic model, the lower ω free vibrations frequency takes the form

$$(\omega^*)^2 = \frac{\breve{B}^*\hat{B}^* - {\bar{B}^*}^2}{\hat{B}^*(\breve{\mu}^* + \breve{\vartheta}^*)}. \tag{5}$$

The results of calculations of lower ω_-^* free vibrations frequency of the tolerance model and lower ω free vibrations frequency of the asymptotic model for thin functionally graded plate will be presented as a summary of the considerations.

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NONLINEAR MODE COUPLING AND INTERNAL RESONANCES IN A PLANAR BEAM-SPRING SYSTEM

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ABSTRACT

We consider an extensible Timoshenko beam-spring system (see Fig. 1). Dynamics of the considered structure is studied by finite element method and analytical multiple time scales method applied to the set of partial differential equations of motion. The analytical model (1)-(5) takes into account geometric nonlinearities, longitudinal, transversal and rotary inertia, as well as elongation and shearing effect of infinitesimally small beam segment (for a more detailed description we refer to [1]). In the analysis we go beyond the Euler-Bernoulli or Timoshenko beam *linear theories* and focus on moderately large *nonlinear free* oscillation and *forced-damped* vibrations, that may result in strong dynamic couplings.The mathematical model is given by partial differential equations:

$$\rho A\ddot{W} + C_W\dot{W} = \left\{EA\left[\sqrt{(1+W')^2+U'^2}-1\right]\frac{1+W'}{\sqrt{(1+W')^2+U'^2}} + GA\left[\theta - \arctan\left(\frac{U'}{1+W'}\right)\right]\frac{U'}{\sqrt{(1+W')^2+U'^2}}\right\}', \tag{1}$$

$$\rho A\ddot{U} + C_U\dot{U} + P_U(Z)\cos(\Omega T) = \left\{EA\left[\sqrt{(1+W')^2+U'^2}-1\right]\frac{U'}{\sqrt{(1+W')^2+U'^2}} - GA\left[\theta - \arctan\left(\frac{U'}{1+W'}\right)\right]\frac{1+W'}{\sqrt{(1+W')^2+U'^2}}\right\}', \tag{2}$$

$$\rho J\ddot{\theta} + C_\theta\dot{\theta} = \left[EJ\frac{\theta'}{\sqrt{(1+W')^2+U'^2}}\right]' - GA\left[\theta - \arctan\left(\frac{U'}{1+W'}\right)\right]\sqrt{(1+W')^2+U'^2}, \tag{3}$$

and boundary conditions at $Z = 0$

$$W = 0, U = 0, M = 0, \tag{4}$$

and at $Z = L$

$$EA\left[\sqrt{(1+W')^2+U'^2}-1\right]\frac{1+W'}{\sqrt{(1+W')^2+U'^2}} + GA\frac{\left[\theta - \arctan\left(\frac{U'}{1+W'}\right)\right]U'}{\sqrt{(1+W')^2+U'^2}} + k_sW = 0, U = 0, M = 0. \tag{5}$$

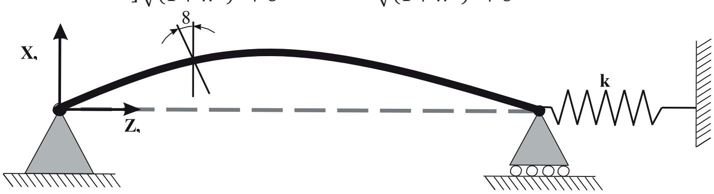

Fig. 1. The planar hinged-simply supported beam with an axial spring k_s. Notation of axial (transversal) $W(U)$ displacements and rotation θ of the cross–section.

In the above formulation a tip inertia or rotary springs studied e.g. in [2] can be easily added, without disturbing the procedure of the analytical solution [3]. It is worth emphasizing that in the recently published book [4], only ordinary differential equations of motion are solved by the multiple time scales method, while in the proposed analytical approach the perturbation method is applied directly to a set of partial differential equations of motion (1)-(3) and associated boundary conditions (4)-(5) without the use of Galerkin discretization.

For *infinitesimally small* amplitudes of oscillations the natural frequency in transversal direction is independent of the axial spring stiffness, but due to sufficient external harmonic excitation in vicinity of natural frequency, the beam undergoes large deformations in nonlinear regime. The axial spring k_s may restrict the movement of the beams' tip, and passively control nonlinear response of the principal resonance [5]. The large amplitudes of flexural oscillations actuate significant movement in the longitudinal direction, which activates higher flexural modes e.g. the first and the second flexural mode with the exchange energy through longitudinal motion as a carrier. This phenomenon is refereed to *3:1 flexural-flexural internal resonance*, accompanied by detached solutions, extra nonlinear peak at main resonant branch or previously unidentified phenomena such as *dynamic buckling* of the beam [6].

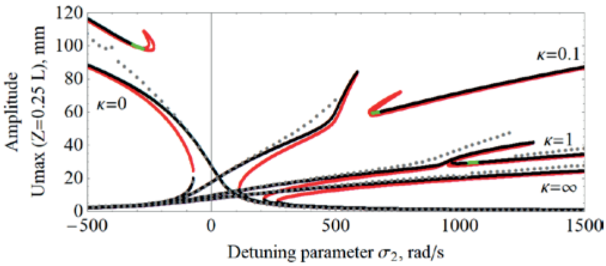

Fig. 2. Frequency response curves: black/green/red lines are stable/unstable source type/ unstable saddle type analytical solution paths and gray dots denote finite element simulations ($\kappa = k_s L/EA$).

The finite element model confirms the analytical predictions, see Fig. 2. For small amplitudes of oscillations the natural frequencies and associated modal shapes coincide, while in nonlinear regime the full frequency response curves are built by *transient in time simulations*. In the future, the problem will be extended to physical boundary conditions suchlike *tip mass* and *rotary inertia of hinges*, and then experimentally validated [3].

Acknowledgments

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COMPARATIVE ANALYSIS OF THE ACOUSTIC PROPERTIES OF GRANULAR MATERIALS

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ABSTRACT

The results of acoustic property tests for six types of granular materials: perlite, vermiculite, active coconut carbon, rubber granulates, pumice and wood chips, which can be used in noise protection structures, are shown in the article. The characteristics of the sound absorption coefficient and the sound transmission loss for material specimens with seven thicknesses in the range of 10-100 mm were determined based on the results of experimental tests carried out with the use of an impedance tube. The relationships between the first resonance frequencies of the sound absorption coefficient and the thicknesses of material specimens were determined. Single-number weighted indices, such as the sound absorption coefficient and the sound reduction index, were determined for the case of normal incidence of a sound wave on a specimen. Subsequently, dependencies of these indices on the surface mass of the tested materials were determined. The research showed that three materials, perlite, vermiculite and active coconut carbon, were distinguished among the examined granules with the best sound-absorbing and sound-insulating properties. Active coconut carbon had the highest value of the sound reduction index.

Acknowledgments

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ANALYSIS OF DYNAMIC CHARACTERISTICS OF THE TURBINE SHAFT VIBRATION IN OXIDIZER TURBO-PUMP DEMONSTRATOR

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ABSTRACT

In the paper, the numerical approach is presented to model the vibration characteristic of the turbo- pump shaft. The modal analysis of turbo-pumps is a crucial part of the mechanical design [1]. The complex geometry of the turbo-pump components introduce difficulties in the determination of modal parameters. In this study, the numerical modal solver (Block Lanczos) was chosen for the extraction of the eigenvalue implemented in Anysys Mechanical [2] with the use of the finite element method. Moreover, in the paper, the simplified rotordynamic model used during the optimization process of the design is described.
The shaft is manufactured with the turbine rotor, to prevent obstacles coming from the imbalance of the shaft assembly (Fig. 1). The 3D-model were also used to prepare the structural numerical mesh used in calculations in Altair Hypermesh. The results of studies indicate changes in the rotor geometry to prevent vibration during load.

Fig. 1. Visualization of turbopump shaft

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USING THE NATURAL MODES OF TRANSIENT VIBRATIONS IN PREDICTIVE MAINTENANCE OF INDUSTRIAL MACHINES

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ABSTRACT

The condition monitoring (CM) of heavy-duty mining and metallurgical machines working under non-stationary loading and cyclic (reversal) speed-changing conditions is always accompanied by transient vibrations [1-3]. The presence of impulsive impacts results in false alarms, which cause either downtime for revisions or unexpected failures and significant production losses. The spatial and torsional vibrations of high amplitude have also resulted from the excessive linear, and angular backlashes, which should be considered as the main parameters for CM since most local defects in gears and bearings are following the dynamic overloads. However, the standard approaches are only partly suitable for backlashes diagnostics and predictive maintenance of such a class of machines. For example, triggering of recording beyond the transients and "angular" approach, i.e. data analysis synchronously with shaft revolutions, are somewhat challenging to implement in machines with a complicated structure.

Some methods are developed for backlashes estimation in the automotive powertrains [4] and rolling mills [5] using transient vibrations. By representing the heavy gearboxes and shafts as multi-body systems [6] with non-smooth stiffness characteristics [7] the diagnostics of bearings wear and bolted joints looseness can be efficiently conducted. In general, modal analysis is a proven method for diagnostics of damages in structures [8,9]. Such features of non-linear vibrations as non-isochronism, delay, the relation of dynamic response and static loads and damping factor can be used for diagnostic purposes. Examples of such machines where this approach is applied are steel rolling mills [10], load-haul-dump vehicles [11] and vibrating machines. Such difficult for diagnostics elements of vibrating screens as supporting springs and bolted joints on sieving decks can be diagnosed based on modal analysis using excitation from external periodical forces of inertial vibrators and stochastic impacts from the pieces of bulk material [12].

The diagnostics based on multi-body dynamical models can be conducted both on time series and in the frequency domain with different metrics (health indicators) derived from the transient signals analysis at the natural modes. The responses calculated on the dynamical models with linear parameters are used as the reference values in diagnosing non-stationary systems with piecewise linear stiffness. Statistical aspects of non-linear systems of machine loading are also important. It is shown that maximum dynamic load and its standard deviation in elastic couplings have a polynomial dependence on static input load.

Based on conducted studies of different industrial machines, we actually propose a new strategy of predictive maintenance, which is characterized for a specific class of applications by the transition from monitoring of local defects to the monitoring of their causes – angular and radial backlashes, which is supported by the appropriate methods of their diagnostics. In addition, the verified models allow the accumulation in the computerized maintenance management systems (CMMS) of the fatigue cycles from dynamic loading in the critical elements of machines where measurement of torques and forces is unavailable [13].

The results of calculations on the multi-body models with non-linear stiffness characteristics are quite different from the FEM calculations conducted at the machine design stage when structural

elements' deterioration is usually not included in the models. The analytical relations of the remain useful life (RUL) of elements with their degradation are shown in the examples.

Acknowledgments
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OPTIMIZATION ANALYSIS OF BAR STRUCTURE INCLUDING NATURAL FREQUENCY

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ABSTRACT

This work is about an optimization process of the steel-concrete observation tower [1]. Using the Dynamo Sandbox software, the parametric geometry model of this structure was created (Fig. 1). On the other hand, the parametric design gives a possibility to establish many models in a short time [2-3]. These models may vary in chosen parameters – the number of bars in selected group or the length of the individual bars or dimensions.

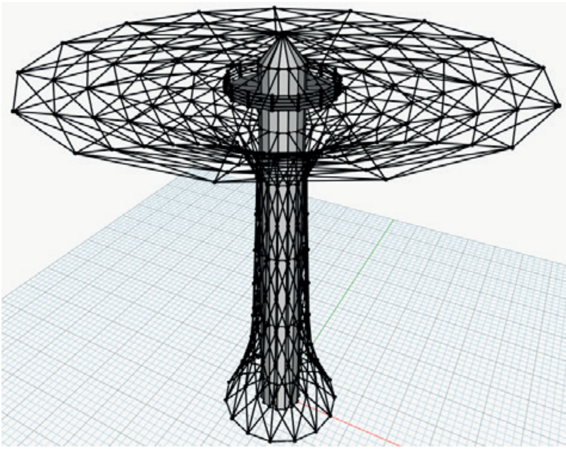

Fig. 1. Geometry model of the structure

Then, the static calculations can be carried out for each of geometry variant and the influence of the chosen parameters on the internal forces and the values of the natural frequencies can be analyzed. In this note the parameters such as the radius of the lower and the upper base, the number of segments of the upper and lower part of the tower, the height of the structure, the number of sides and the radius of the viewing gallery were analyzed.
By adopting the chosen optimization criterion, it is possible to choose the optimal variant of the geometry and adapt its shape and dimensions to the requirements.

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ACOUSTICALY IMPROVED POLISH PHARMACY ROBOT FABLOX

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ABSTRACT

This work presents the results of acoustical improvements of a Polish pharmacy robot Fablox (Fig. 1.) The first vibroacustical analysis of a Polish pharmacy robot was made in a previous research work [1], which determined the initial parameters of vibrations and noise. On their basis, guidelines for the reduction of vibrations and noise were developed. Among other things, the vibration isolation of drives and structural elements was used, the casing was sealed, and the robot's interior was lined with sound-absorbing materials. The end result of these actions allowed to obtain a quiet pharmacy robot that does not adversely affect the acoustic climate in the pharmacy room and ensures proper working conditions for pharmacists.

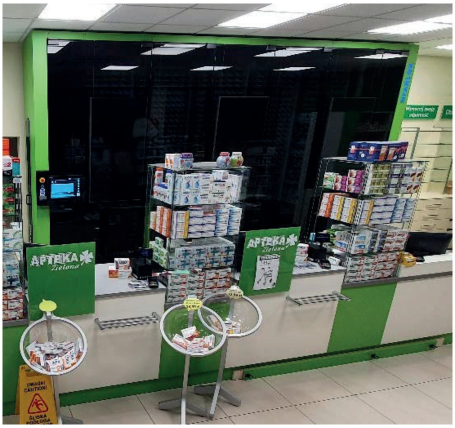

Fig. 1. Acoustically improved Polish pharmacy robot Fablox.

Acknowledgments

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30th Conference
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APPLICATION OF ISOGEOMETRIC APPROACH TO DYNAMICS OF CURVED BEAMS

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ABSTRACT

Isogeometric analysis was introduced by Hughes et al [1,2] in 2005 as a bridge between Computer Aided Design (CAD) and finite element analysis (FEA). In recent years, a lot of interest in this method can be observed. The fundamental difference between the isogeometric approach and FEA is that the first one uses B-splines instead of the interpolation functions used in FEA. In this case, it is easier to carry out an analysis using basis functions of different order and introduce refinement schemes than in the case of the classical FEA.

The advantage of using the isogeometric approach can be seen in the case of the analysis of curved structures, where it is possible to accurately describe the geometry and displacements of the considered system. An analysis of straight and curved beams in the isogeometric approach can be found in [3].

The analysis of free vibrations of curved beams with different boundary conditions and different geometry will be presented. The obtained solutions using the isogeometric approach [4] will be compared with the solutions obtained using FEA and available in the literature.

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30th Conference
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LABORATORY STAND OF CHAIN CONVEYOR

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ABSTRACT

Chain conveyors are short distance transport devices that are commonly used in industry. In the literature one can find recommendations according to which these devices should be designed. Mainly provided are the procedures for the drive of devices, including sprockets. The design of the chain wheel affects the speed distribution of the chain elements during the conveyor operation. This determines the parametric vibrations of the chain. In extreme cases, these vibrations may damage the structure of the device.

The paper describes a laboratory stand for the chain conveyor model (Fig. 1), which is to be used, inter alia, for educational purposes. The stand was constructed in such a way that it was possible to change the operational parameters of the conveyor, e.g. drive speed and chain tension. Measurements on the real object help in the parameterization of the mathematical model describing the dynamic phenomena related to the operation of the conveyor.

Fig. 1. Chain conveyor laboratory stand

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OPTIMAL VIBROISOLATION OF MECHANICAL PRESS SUBJECTED TO POLYHARMONIC EXCITATION

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ABSTRACT

The paper deals with the problem of vibroisolation of a mechanical press (Fig. 1) subjected to polyharmonic inertial excitation. In computational model small vibrations of a linear system performing a planar motion were taken. It was assumed that the physical parameters describing the stiffness and damping of the supports on the left and right side of the machine may be different.
The aim of the calculations is to find the parameters of the supports for which the minimum value of the introduced optimization indicator, dependent on the forces transferred to the ground in the vertical and horizontal directions, is obtained.
Although the considered mechanical system is relatively simple, the optimization results depend on the value of the fundamental frequency, the form of the polyharmonic excitation spectrum and the adopted frequency range in which the vibration isolation should be effective.
Using the results of numerical calculations, the different solutions to the problem of vibroisolation of the machine are presented, considering harmonic components of the force transmitted to the floor.

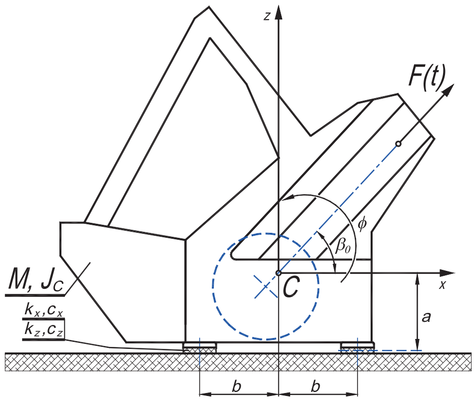

Fig. 1. Mechanical press subjected to inertial polyharmonic excitation

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FREE FLEXURAL VIBRATIONS OF AN EXPANDED-TAPERED SANDWICH BEAM

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ABSTRACT

The subject of the paper is a simply supported expanded-tapered sandwich beam of length L (Fig.1).

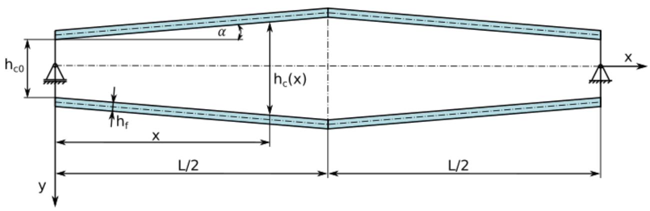

Fig. 1. Scheme of the expanded-tapered sandwich beam

The simplified analytical model of this beam is elaborated taking into account the paper [3] with omitting the shear effect. Based on the Hamilton's principle, the differential equation of motion of this beam was obtained in the following form

$$\left[\sqrt{e_c} f_\alpha(\xi) + k_f\right]\frac{\partial^2 v}{\partial t^2} + \left[f_{v4}(\xi)\frac{\partial^4 v}{\partial \xi^4} + f_{v3}(\xi)\frac{\partial^3 v}{\partial \xi^3} + f_{v2}(\xi)\frac{\partial^2 v}{\partial \xi^2}\right]\frac{E_f}{12\varrho_f \lambda^2 L^2} = 0. \quad (1)$$

where: $f_{v4}(\xi) = e_c f_\alpha^3(\xi) + 3f_\alpha^2(\xi)k_f + 3f_\alpha(\xi)k_f^2 + k_f^3$, $f_{v2}(\xi) = 24\lambda^2\left[e_c f_\alpha(\xi) + k_f\right]tan^2\alpha$, $f_{v3}(\xi) = 12\lambda\left[e_c f_\alpha^2(\xi) + 2f_\alpha(\xi)k_f + k_f^2\right]tan\alpha$, $f_\alpha(\xi) = 1 + 2\lambda\xi tan\alpha$, $k_f = 2\chi_f/cos\alpha$, $\chi_f = h_f/h_{c0}$, $\lambda = L/h_{c0}$, $\xi = x/L$, $e_c = E_c/E_f$, $\sqrt{e_c} = \varrho_c/\varrho_f$, $E_f, E_c, \varrho_f, \varrho_c$ Young's modules and mass densities of the faces and core, $v(\xi,t) = \bar{v}(\xi)v_a(t)$ – the deflection line.

Taking into account the paper [4], the equation of motion (1) was analytically solved with consideration of the deflection line of this beam subjected to its own weight. Therefore, the dimensionless intensity of the load is as follows

$$\bar{m}(\xi) = \sqrt{e_c} f_\alpha(\xi) + k_f. \quad (2)$$

Consequently, the dimensionless bending moment

$$\bar{M}(\xi) = \frac{1}{12}\left[6\left(\sqrt{e_c} + k_f\right)(\xi - \xi^2) + \sqrt{e_c}\lambda(3\xi - 4\xi^3)tan\alpha\right]. \quad (3)$$

Thus, the dimensionless deflection line of this beam is as follows

$$\bar{v}(\xi) = C_1\xi - \iint \frac{\bar{M}(\xi)}{f_{v4}(\xi)} d\xi^2 . \quad (4)$$

where integration constant $C_1 = \int_0^{1/2} \frac{\bar{M}(\xi)}{f_{v4}(\xi)} d\xi$.

The differential equation of motion (1) is approximately solved with used the Galerkin method. Substituting the function (4) into the equation (1), and after simply transformation, one obtains the equation

$$\frac{d^2 v_a}{dt^2} + \frac{J_v}{J_k} \frac{E_f}{12\varrho_f \lambda^2 L^2} v_a(t) = 0\,, \tag{5}$$

where: $J_v = \int_0^{1/2} \left[\sqrt{e_c} f_\alpha(\xi) + k_f\right] \bar{v}(\xi) d\xi$, $J_k = \int_0^{1/2} \left[\sqrt{e_c} f_\alpha(\xi) + k_f\right] \bar{v}^2(\xi) d\xi$.

This equation (5) is solved with the use of the function $v_a(t) = v_a sin(\omega t)$, where: v_a – amplitude of the flexural vibration and ω – fundamental natural frequency.

Substituting this function into the equation (5) one obtains the fundamental natural frequency in the following form

$$\omega = \frac{\sqrt{3}\,10^3}{6\lambda L} \sqrt{\frac{J_v\, E_f}{J_k\, \rho_f}} \left[\frac{rad}{s}\right], \quad \text{or} \quad f_z = \frac{\omega}{2\pi} = \frac{\sqrt{3}\,10^3}{12\pi\lambda L} \sqrt{\frac{J_v\, E_f}{J_k\, \rho_f}} \;\; [Hz]. \tag{6}$$

Exemplary calculations are carried out for the following data of the beam (Fig.1): Young's modules E_f=72 GPa, E_c=2.25 GPa, mass densities ρ_c=2710 kg/m3, ρ_f=479 kg/m3 and h_{c0}=16 mm, h_f=2 mm. The values of the fundamental natural frequency are analytical and numerical FEM calculated for exemplary two beams of length L=0.6 m and 0.8 m and graphical presentation in Figure 2.

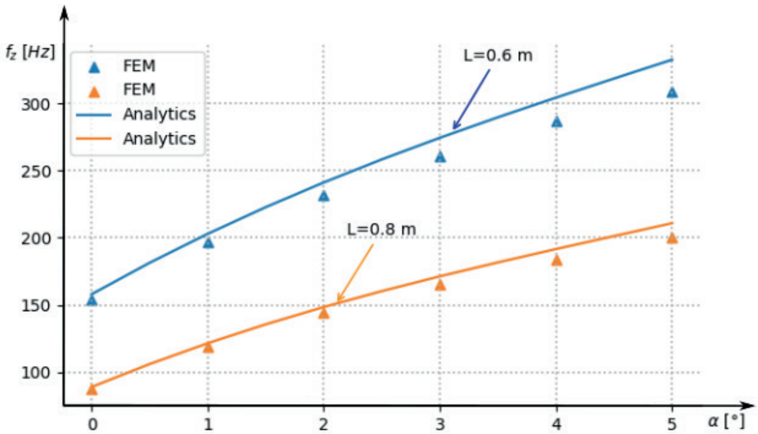

Fig.2. Graphs of the fundamental natural frequency of two example beams

The differences between fundamental natural frequencies values (Fig.2), determined by two methods, are less than 7%. The flexural vibrations problem of the expanded-tapered beams has been the subject of research for many years and is presented, for example, in papers [1] and [2].

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FREE FLEXURAL VIBRATIONS OF A SANDWICH BEAM ON AN ELASTIC FOUNDATION WITH VARIABLE PROPERTIES

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ABSTRACT

The subject of the paper is a simply supported sandwich beam on an elastic foundation with variable properties (Fig.1).

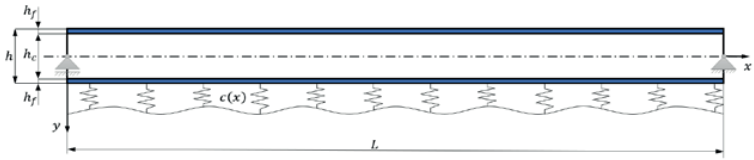

Fig. 1. Scheme of the sandwich beam on the elastic foundation

Taking into account the paper [4], two differential equations of motion are formulated in the following form

$$\rho_f C_m \frac{\partial^2 \bar{v}}{\partial t^2} + \frac{E_f}{12\lambda^2 L^2}\left(C_{vv}\frac{\partial^4 \bar{v}}{\partial \xi^4} - C_{v\psi}\frac{\partial^3 \psi_f}{\partial \xi^3}\right) + \frac{1}{bh}c(\xi)\bar{v}(\xi) = 0, \tag{1}$$

$$C_{v\psi}\frac{\partial^3 \bar{v}}{\partial \xi^3} - C_{\psi\psi}\frac{\partial^2 \psi_f}{\partial \xi^2} + \lambda^2 C_\psi \psi_f(\xi, t) = 0, \tag{2}$$

where: $C_m = 1 - \left(1 - \sqrt{e_c}\right)\chi_c$, $C_{vv} = 1 - (1 - e_c)\chi_c^3$, $C_{v\psi} = 3 - (3 - 2e_c)\chi_c^2$, $\chi_c = \frac{h_c}{h}$, $C_{\psi\psi} = 4[3 - (3 - e_c)\chi_c]$, $C_\psi = \frac{24}{1+\nu_c}\frac{e_c}{\chi_c}$, $c(\xi) = c_{fd}\left[1 + \alpha_f sin^n(\pi\xi)\right]$, $\frac{\varrho_c}{\varrho_f} = \sqrt{e_c}$, $e_c = \frac{E_c}{E_f}$,
$\xi = x/L$ – dimensionless coordinate, α_f – coefficient, c_{fd} $[\mathrm{N/mm^2}]$ – elastic foundation constant, $n = 1,2,3 \ldots$ – exponent, $E_f, E_c, \varrho_f, \varrho_c$ – Young's modules and mass densities of the faces and the core, $\bar{v}(\xi, t) = v(\xi, t)/L$ – dimensionless deflection, $\psi_f(\xi, t)$ – displacement function, t – time.
These two differential equations of motion are approximately solved with the use of two assumed functions

$$\bar{v}(\xi, t) = [\sin(\pi\xi) + k_v \sin(3\pi\xi)]\bar{v}_a(t), \quad \psi_f(\xi, t) = \left[\cos(\pi\xi) + k_\psi \cos(3\pi\xi)\right]\psi_{fa}(t), \tag{3}$$

where: k_v, k_ψ – dimensionless coefficients.
Substituting these functions into the equation (2), and after simply transformations, one obtains

$$k_\psi = 27\frac{\pi^2 C_{\psi\psi} + \lambda^2 C_\psi}{9\pi^2 C_{\psi\psi} + \lambda^2 C_\psi}k_v, \quad \psi_{fa}(t) = \frac{\pi C_{v\psi}}{C_{\psi\psi} + (\lambda/\pi)^2 C_\psi}\bar{v}_a(t). \tag{4}$$

The differential equation of motion (1), solved with the used the Galerkin method, has the form

$$\varrho_f C_m \frac{d^2\bar{v}_a}{dt^2} + \frac{\pi^4 E_f}{12\lambda^2 L^2}\left[(1+81k_v^2)C_{vv} - \left(1+27k_v k_\psi\right)C_{se} + \frac{24}{\pi^4}\lambda^3\frac{L}{b}J_n\frac{c_{fd}}{E_f}\right]\frac{\bar{v}_a(t)}{1+k_v^2} = 0, \qquad (5)$$

where $C_{se} = C_{v\psi}^2/\left[C_{\psi\psi} + (\lambda/\pi)^2 C_\psi\right], J_n = \int_0^1 \left[1+\alpha_f \sin^n(\pi\xi)\right]\left[\sin(\pi\xi) + k_v \sin(3\pi\xi)\right]^2 d\xi$.
The equation (5) is solved with the use of the following function $\bar{v}_a(t) = \bar{v}_a \sin(\omega t)$, where $\bar{v}_a$ – amplitude of the flexural vibration, ω [rad/s] or f_z [Hz] $= \omega/2\pi$ – fundamental natural frequency. Thus, substituting this function into the equation (5) one obtains

$$\omega = \min_{k_v}\left\{\frac{\sqrt{3}\pi^2 10^3}{6\lambda L}\sqrt{\left[(1+81k_v^2)C_{vv} - \left(1+27k_v k_\psi\right)C_{se} + \frac{24}{\pi^4}\lambda^3\frac{L}{b}J_n\frac{c_{fd}}{E_f}\right]\frac{1}{1+k_v^2}\frac{E_f}{C_m\varrho_f}}\right\}. \qquad (6)$$

The detailed studies are carried out for exemplary sandwich beam on the elastic foundation for the following data: dimensions $L = 0.3$ m, $h = 20$ mm, $h_c = 18$ mm, $b = 20$ mm, the material constants $E_f = 72$ GPa, $E_c = 3.0$ GPa, $\rho_f = 2710$ kg/mm^3, $\rho_c = 553$ kg/mm^3, $\nu_c = 0.3$, the elastic foundation constant $c_{fd} = 8$ MPa. The results of the analytical and numerical FEM calculations for the sandwich beam on exemplary elastic foundation with variable properties are specified in Table 1.

Table 1. The values of the fundamental natural frequency of exemplary sandwich beam

	α_f	-0.50	-0.25	0	0.25	0.50
$n = \mathbf{1}$	k_v	-0.003966	-0.001989	0	0.001996	0.004001
	$f_z^{(An)}$ [Hz]	804.40	887.16	962.77	1032.81	1098.34
	$f_z^{(FEM)}$ [Hz]	801.17	883.52	958.73	1028.4	1093.5
$n = \mathbf{2}$	k_v	-0.005832	-0.002925	0	0.002941	0.005901
	$f_z^{(An)}$ [Hz]	824.23	896.25	962.77	1024.87	1083.30
	$f_z^{(FEM)}$ [Hz]	821.04	892.63	958.73	1020.4	1078.5

The FEM model of the sandwich beam on exemplary elastic foundation with variable properties is developed with the use of the SolidWorks system. The differences between analytical and numerical FEM values of the fundamental natural frequency do not exceed 0.44 %. The flexural vibration problem of beams on elastic foundations is presented for example in papers [2] and [3].

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DETERMINATION OF SOUND INSULATION PROPERTIES OF HOMOGENEOUS BAFFLES USING FINITE ELEMENT METHOD

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ABSTRACT

The basic parameter of materials used in constructional solutions of anti-noise protection, is sound insulation, which can be determined in laboratory conditions and also using theoretical models. The use of numerical methods in the form of the Finite Element Method to calculate the mechanical impedance of a baffle and then the sound insulation of homogeneous baffles was presented in the article. A 1 mm thick steel plate with a square, rectangular and round shape was analyzed. The boundary conditions for simply supported and clamped plate were taken into account in the numerical calculations. The results of the calculations were compared to both the commonly used the mass law and to the experimental tests. These analyzes will be the starting point for analyzes of multi-layer baffles, for which it is no longer possible to apply the mass law.

Acknowledgments
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NATURAL FREQUENCIES HINGED ALONG THE LOWER RIBS OF THE TRANSVERSAL-ORTHOTROPIC PLATE-STRIP

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ABSTRACT

Plate-strips of transversely orthotropic materials make up a significant proportion of building structures, as well as devices and technical means for various purposes [4]. The effect of cyclic loads on them necessitates a reliable determination at the design stage of the natural frequency spectrum in order to prevent resonant phenomena in operating conditions. Their values significantly depend on the method of fixing the side elongated faces of the plate-strips. In the case of fixing them on the lower elongated ribs, it is impossible to build a computational model using both the classical Kirchhoff-Lev plate theory and the generalized theory based on the sliding model of S. P. Timoshenko [3]. Therefore, the proposed work considers a variant of the refined theory of dynamic deformation of transversal-orthotropic plates, which approximates all the characteristics of their spatial stress-strain state [1, 2]. This allowed us to formulate a correct initial-boundary value problem about the free vibrations of the above-mentioned plate-strips.
Expressions for natural frequencies are obtained. The nature of their dependence on the parameters of transverse orthotropic, in particular susceptibility to transverse compression, analyzed. For the case of isotropy of elastic characteristics of the plate-band material the values of natural frequencies are compared with those obtained using classical and generalized theories in the conventional recording of the conditions of hinged fixation of the side faces [3]. The coincidence of the results obtained according to the proposed new calculation scheme and the two-dimensional relations of the dynamic theory of elasticity using the finite element method is noted.

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TOLERANCE MODELLING OF VIBRATIONS OF A SANDWICH PLATE WITH A HONEYCOMB CORE

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ABSTRACT

In this paper the vibrations of a simply supported sandwich plate are investigated. In general sandwich plates are made of two outer layers (so called *faces*) connected with each other with a core, cf. Fig. 1. By introducing several additional assumptions, such as: the symmetric form of the plate towards its midplane, an isotropic nature of all materials, which create the whole structure, and broken line de- formation hypothesis, the vibrations of the plate can be defined by a system of well-known partial differential equations. The exact form and description of those equations can be found in literature, cf. [1–3].

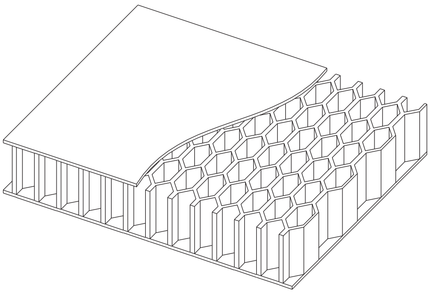

Fig. 1. Sandwich plate with a honeycomb core

Let us now consider a special case, where the core of the sandwich plate takes the form of a honey- comb structure. In such case the core can be treated as a layer of periodically distributed isotropic materials. As a result, the governing equations of the sandwich plate constitute a system of partial differential equations with periodic, non-continuous coefficients. The exact analytical solution to such defined system is very difficult to obtain. However, its approximated solution can be easily derived by the application of *the tolerance averaging technique*, cf. [4, 5]. Following the tolerance modelling procedure, the governing equations are averaged on a basic repeatable element, called *periodicity cell*. Eventually, after a series of tolerance averaging approximations, the most convenient form of the av- eraged governing equations, characterised by constant coefficients, is obtained.

In this paper the derived tolerance model of sandwich plate is used to estimate the free vibration fre- quencies of a specific set of exemplary simply supported structures with honeycomb core. The anal- ysed structures vary from each other with shape and thickness of the honeycomb core and the charac- teristic dimensions of the whole plate. The obtained results are compared with a benchmark solutions of FEM models. Basing on their convergence the applicability of the proposed tolerance model in the vibration analysis is assessed.

Acknowledgments
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RESPONSE OF LATTICE STRUCTURES BASED ON THE TRIPLY PERIODIC MINIMAL SURFACES TO PROJECTILE IMPACT

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ABSTRACT

Lattice structures based on the Triply periodic minimal surfaces (TPMS) are recently of significant interest because of their specific mechanical properties and low weight. Most studies focus on the static or dynamic compression of these structures [1]. The goal of this paper is to verify the response of selected TPMS lattices to projectile impact. The design of such structures is not easy because of their complex shapes and thus a special freely-available software called MSLattice [2] was used to prepare the geometry for numerical analyses. Two types of TMPS shapes were selected for this study – gyroid and I-WP. The analyses were performed using an explicit dynamics solver in Abaqus FEA software, assuming a rigid projectile. Johnson-Cook plasticity model was used, including failure and element deletion [3]. Exemplary results from the analyses are presented in Fig. 1.

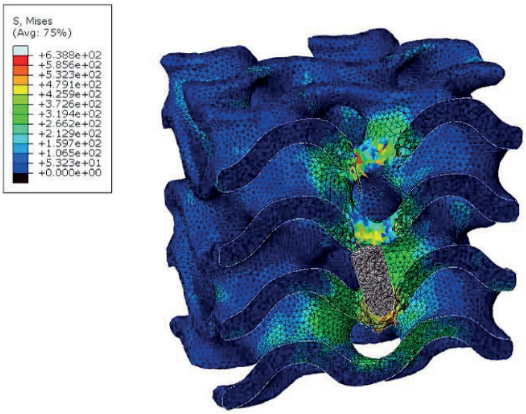

Fig. 1. Section view of von Mises stress in gyroid lattice after projectile impact

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THE EXPERIMENTAL STAND FOR OBSERVATION AND CONTROL OF DYNAMICS OF AN EXTENDED ATWOOD'S MACHINE

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ABSTRACT

The original experimental stand presented here for the first time is shown in Fig. 1. It has been constructed to observe and verify dynamical behavior of a swinging Atwood machine studied in [1] and extended in [2]. The laboratory stand consists of a frame, a Rexroth servo-motor with driver and software, two suspension pulleys, where one is movable with the use of an arm mounted on the motor shaft and one is fixed, a flexible string with two pendulum masses fixed in a distance and a counterweight mass on the second end of the string subjected to forcing from two electromagnets.

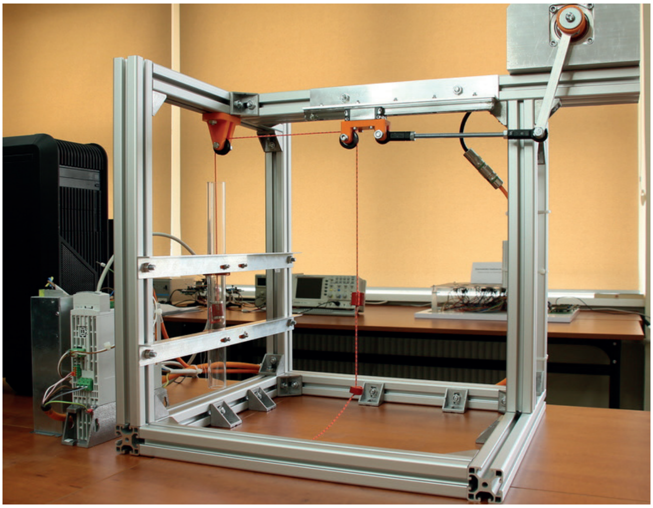

Fig. 1. The experimental stand

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APPLICATION OF SUPERVISED LEARNING ALGORITHMS FOR ANALYSIS THE VIBRATIONS OF AN OSCILLATOR FORCED BY A RANDOM SERIES OF IMPULSES

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ABSTRACT

The vibrations of a discrete, linear system, excited by stochastic impulses with the following initial conditions $x(0)=0$ and $dx/dt=0$ are described by the equation [1]

$$\frac{d^2x}{dt^2}+2b\frac{dx}{dt}+a^2x=\sum_{t_i<t}\eta_i\delta(t-t_i) \tag{1}$$

where $i=1,2,3\ldots$ is a number of impulse,
η_i is a sequence of independent identically random values of amplitude of i-th impulse with finite expectation, and it assumes a finite number of values $p_i=p(\eta_i)$.
t_i –i-th time of excitation of the vibrations. The intervals between the impulses t_{i-1} - t_i is a sequence of independent identically randomly distributed variables with exponential distribution with the impulse rate λ. The intervals between the impulses and amplitudes of the impulses are independent random variables.

In this case the mathematical model of an oscillator is understood in accord with the terminology used for defining statistical models as a formalized description of a certain theory or causal situation that are assumed to generate the observed data. In this article we introduce application of supervised learning algorithms to solve the problem of recognizing the distribution of impulses. Inverse problem, distributions of probabilities computed on the basis of a single sample for the finite time interval are always burdened with uncertainty. The time series should be described in terms of statistical properties. However, this should be taken into account when designing the experiment that the random vibrations of a discrete system depend on parameters connected with oscillators (damping and the frequency of damped vibrations) as well as the parameters characterizing the random series of impulses (intensity of impulse occurrence, values of impulses, distributions of impulse values). Among the infinite number of possibilities there exist only a few types of oscillators with very strong damping and high frequency of damped vibrations that can be applied to generate the data that can be used in machine learning algorithms.

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APPLICATION OF UNSUPERVISED LEARNING ALGORITHMS FOR ANALYSIS THE VIBRATIONS OF AN OSCILLATOR FORCED BY A RANDOM SERIES OF IMPULSES

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ABSTRACT

The vibrations of a discrete, linear system, excited by a random series of impulses described by the equation

$$x(t) = \frac{1}{c} \sum_{0<t_i<t} \eta_i e^{-b(t-t_i)} \sin(c(t-t_i)) \tag{1}$$

where η_i is a sequence of independent identically random values of amplitude of i-th impulse with finite expectation.
The time intervals between impulses $\tau_i=(t_{i-1} - t_i)$ are independent continuous random variables for which the function of probability density assumes the form of exponential distribution (2).
The intervals between the impulses is a sequence of independent identically randomly distributed variables with exponential distribution with the impulse rate λ.
A mathematical model the inverse identification problem that allows for determining the distribution of value of impulses forcing the vibrations of the system was developed in several stages and was constructed on the basis of linear differential equations [1] using the ergodic theory together with the basics of the theory of dynamic systems, measure theory, group theory, probability calculus and the theory of stochastic processes based on it, and when $t \to \infty$.

At the present level of knowledge, execution of an experiment in which Dirac Delta would occur at the impulse and the restitution coefficient would not be necessarily taken into account in the model is practically impossible. However, it is possible to execute simulated studies, which are an approximation of the modeled phenomenon, and in these studies the qualitative analysis is used to prepare the experiments in the proper way. It should be remembered that $x(t)$ for t <3600 seconds is a time series that can be described in terms of statistical properties. In this article we introduce application of unsupervised learning algorithms to solve the problem of recognizing the distribution of impulses generated for different distributions values of impulses.

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MODEL OF THE MANIPULATOR WITH FLEXIBLE JOINTS

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ABSTRACT

Vibration analysis of industrial robots is one of the key issues in the context of robotization of machining processes. Robot vibrations during the machining process negatively affect both the robot's mechanisms and the workpiece. Low-frequency vibrations, resulting from the flexibility in the manipulator joints and the tool mounting system, mainly reduce the dimensional and shape accuracy of the workpiece. As part of the work, a mathematical model of a two-link manipulator was formulated in the form of dynamic equations of motion. The influence of the robot arm configuration on vibration phenomena was investigated. The frequency spectrum of the vibrations of the robot links in many configurations was determined. Obtaining a map of resonant frequencies depending on the manipulator configuration (Fig. 1). Experimental studies were carried out with the use of the IRB1600 robot, which confirmed the conclusions of the simulation tests and enabled the evaluation of the mathematical model.

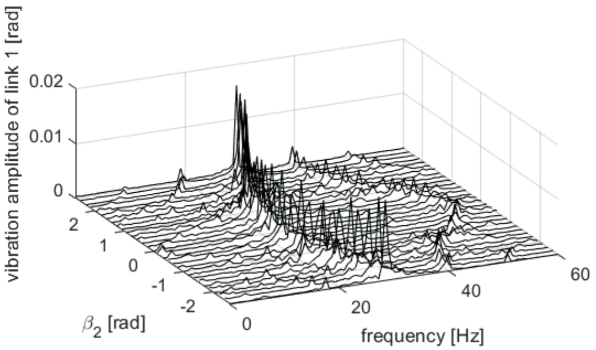

Fig. 1. Map of resonance frequencies - vibrations of the link 1

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AN OVERVIEW OF NONLINEAR VIBRATION PHENOMENA IN HYDRODYNAMIC JOURNAL BEARINGS

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ABSTRACT

Hydrodynamic journal bearings are widely used in the field of rotational machinery for their low friction behaviour, low wear and vibration-damping capabilities. However, when hydrodynamic journal bearings are used to support a rotor, the whole system becomes a complex dynamic system that may exhibit fluid-induced instabilities. Understanding of behaviour of the journal bearing closely before, during and after the instability origin and growth is the main motivation for complex research of local and global dynamics of rotor-bearing systems.

Fluid film bearing related characteristics such as self-excited oscillations, thermo-elastic expansion, and fluid inertia are known to induce nonlinear response states of the rotor system such as sub-synchronous, quasiperiodic, aperiodic, end even chaotic motion.

There are several common methods for the modelling and dynamical analysis of structural parts of rotating systems. A standard possibility of the dynamical modelling of rotating bodies is based on the finite element method considering one-dimensional Euler-Bernoulli or Timoshenko beams. The models respect the continuous mass of rotating shafts and the possible effects of lumped masses such as discs, gear wheels, etc. The mathematical model is derived in the form of the second-order ordinary differential equation (ODE). Another possibility is the utilization of approaches based on the multibody dynamics. The vibrations and global motion of each flexible body are described by a system of differential-algebraic equations (DAE). Elasto-hydrodynamic forces acting in radial journal bearings are computed from bearings load, which can be expressed as an integral of fluid pressure over the bearing surface. Hydrodynamic fluid pressure is computed employing the Reynolds equation, a partial differential equation, which can be derived from the Navier-Stokes equations [1]. The Reynolds equation is used in various modifications (e.g. [2]) and can consider surface roughness, contact interaction, friction effects, etc.

The hydrodynamic forces acting in journal bearings are inherently nonlinear and can induce several types of nonlinear phenomena [3]. The best-documented nonlinear phenomenon occurring in the journal bearings is oil whirl/whip [4]. Oil whirl can be predicted using so-called dynamic coefficients, which describe the linear relation between the hydrodynamic forces with journal displacements and velocities around its equilibrium [5]. Oil whirl occurs after the journal surpasses the threshold speed and loses linear stability [4]. The unstable journal would oscillate with an exponentially growing magnitude. However, its motion is restricted by the bearing clearance in real-world bearings, and hence dynamics beyond the threshold speed cannot be predicted by the linear model [6]. If the oil whirl frequency coincides with a shaft's natural frequency and becomes locked into it, the instability is known as oil whip (e.g. [7]). Oil whip self-excited vibrations are generally unstable and potentially destructive. However, some systems exhibit stable whip self-excited vibrations. In such a case, the increase of the shaft speed leads to further destabilization and the second mode whirl occurs [7]. Some sub-synchronous oscillations can also appear if a flexible rotor

passes through the weekly damped critical speed. These oscillations have the character of a stable whip and quickly disappear with the rising speed.
Another type of nonlinear phenomenon stems from an interaction between the hydrodynamic forces and forces due to rotating unbalance [3]. Barrett et al. [8] were one of the first who studied this interaction in 1976. They discovered that the rotating unbalance could suppress vibration due to the oil whirl. Some twenty years later, Brown et al. [9] found that the rotating unbalance could induce chaotic vibration.
Other nonlinear phenomena have connections with thermal effects [10] and friction or contact forces in self-equalising bearings [3]. Non-uniform shear stress in the oil film can heat the journal surface due to the hydrodynamic friction. If only a small part of the surface is heated repeatedly, a shaft bends, changing the unbalance distribution. This thermal instability is called the Morton effect [10]. Although the sub-synchronous vibrations are dominated by excessive squeezing of the oil film and pad to journal contacts [11], the bearing susceptibility to pad fluttering is greatly influenced by friction between movable components [3].
Textured surfaces (roughness, grooves, discrete textures and grooves) for contact performance enhancement in terms of load-carrying capacity, film thickness, friction and wear are often used in hydrodynamic journal bearings. A variety of different numerical models have been employed in order to find optimal texturing parameters (shape, size, distribution) [12]. Although promising results have been obtained, finding optimal texturing parameters is still very challenging due to the large number of variables involved.

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ON MITIGATION OF OSCILLATIONS OF A MECHANICAL SYSTEM WITH TWO DEGREES OF FREEDOM IN THE VICINITY OF EXTERNAL RESONANCES

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ABSTRACT

Due to the constant interest in expanding the performance range of engineering systems, accounting for non-linear components is increasingly being used in real-world applications. Therefore, the weakening of resonant oscillations of nonlinear structures becomes a task of great practical importance [1 – 3]; this is the subject of the present study. This presents a serious problem, as non-linear systems exhibit rich and complex phenomena that linear systems cannot. In particular, one of the key characteristics of nonlinear oscillations is that their frequency is inseparably dependent on the amplitude of motion.

In present study we analyze the dynamics of 2-DoF nonlinear mechanical system under the influence of an external harmonic force. The system consists of a nonlinear oscillator (primary mass) and attached linear dynamic absorber. The problem of using an analytical approach for tuning the parameters of the absorber in order to neutralize the increase in the amplitude of forced oscillations of the main system in the vicinity of the peaks of the frequency-amplitude curve is considered. It has been shown that when using a linear absorber connected to a Duffing oscillator, the choice of proper stiffness and damping ratios gives acceptable results. An important circumstance is the dependence of stiffness of the absorber not only on the linear component of the stiffness of the main mass, but also on the nonlinear component. Note also that, in contrast to the linear system, the stiffness ratio must exceed the value 1. Also the stability conditions of the stationary regimes of the averaged equations are obtained analytically. These regimes correspond to the quasi-periodic motions of the original input system.

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SYNCHRONIZATION AND ENERGY TRANSFER IN 4DOF FRICTION-INDUCED SELF- AND PARAMETRICALLY EXCITED OSCILLATORS

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ABSTRACT

We consider a practical case in which parametric excitation is provided by a massless rotor of rectangular cross-section with a cylinder-like mass concentrated at the center. The rotor arrangement is placed on a friction-induced self-excited support in the form of a frame placed on a belt moving with constant velocity. This frame is connected to a supplementary mass. A Stribeck friction model is considered for the mass in contact with the belt. The frictional force between the mass and the belt is oscillatory in nature because of the variation of normal force due to parametric excitation from the rotor.

The model shown in Fig. 1 is an expanded form of the one in which zones of instability were studied by Authors in [1]. Studies in [2–3] dealt with synchronization, chaos and other phenomena which are based on continuous system of lesser degrees of freedom. In this work, we show the effect of friction-induced self-excitation and how it interacts with the parametric excitation of the rotating mass. Synchronization has been identified as one of the features, and energy transfer between the masses which leads to this effect have been studied.

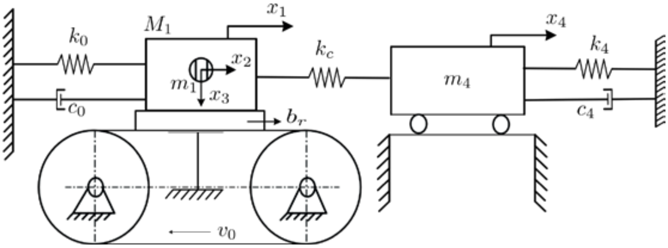

Fig. 1. Model of rotor with parametric excitation placed on friction-induced self-excited support, connected to a supplementary degree of freedom

Equation of motion

The governing equation of motion for the system has been derived for which, in nondimensional form, it is given in Equation (1)-(4):

$$\ddot{X}_1 = -X_1(A\gamma^2 + A_1\gamma^2 + A_2\gamma^2\cos(2\gamma\tau)) + X_2(A_1\gamma^2 + A_2\gamma^2\cos(2\gamma\tau)) - X_3(A_2\gamma^2\sin(2\gamma\tau)) + X_4 D\gamma^2 - \dot{X}_1 h_1\gamma + [X_1(A_2\gamma^2\sin(2\gamma\tau)) - X_2(A_2\gamma^2\sin(2\gamma\tau)) + X_3(A_1\gamma^2 - A_2\gamma^2\cos(2\gamma\tau)) + 1]b_r \tag{1}$$

$$\ddot{X}_2 = X_1(b_1\gamma^2 + b_2\gamma^2\cos(2\gamma\tau)) - X_2(b_1\gamma^2 + b_2\gamma^2\cos(2\gamma\tau)) + X_3(b_2\gamma^2\sin(2\gamma\tau)) + \kappa\gamma^2\sin(\gamma\tau + \phi_0) \tag{2}$$

$$\ddot{X}_3 = -X_1(b_2\gamma^2\sin(2\gamma\tau)) + X_2(b_2\gamma^2\sin(2\gamma\tau)) - X_3(b_1\gamma^2 - b_2\gamma^2\cos(2\gamma\tau)) + \kappa\gamma^2\cos(\gamma\tau + \phi_0) + 1 \tag{3}$$

$$\ddot{X}_4 = X_1 d\gamma^2 - X_4 - \dot{X}_4 h_4\gamma \tag{4}$$

where $A_{1,2}, b_{1,2}$ represent the sum and difference of the excitation frequencies, b_r – dry friction, $h_{1,4}$ – damping, while d – stiffness of the coupling spring, and $0 < \gamma < 1$.

Results

Mass m_4 is of practical importance and as such, we consider its displacement with respect to other structures (M_1). Synchronization between the two stems from complex pattern due to the present of higher harmonics to simple one at m-periodic windows (Fig. 2(c)). This effect reduces the self-excitation such that the M_1 behaves in a manner that is congruent to the slipping phase. Fig. 2 (a) shows its phase portrait and Poincare section at $\gamma = 0.22$ while Fig. 2 (b), (d) show the bifurcation diagram and the average power in the system. There are creation and destruction of limit cycles in the interval $0.1 < \gamma < 0.3$ leading to Neimark-Sacker bifurcation at $\gamma = 0.295$. In same vein, the energy plots show that the kinetic energy of m_4 is highest at the point greatest point of absorption.

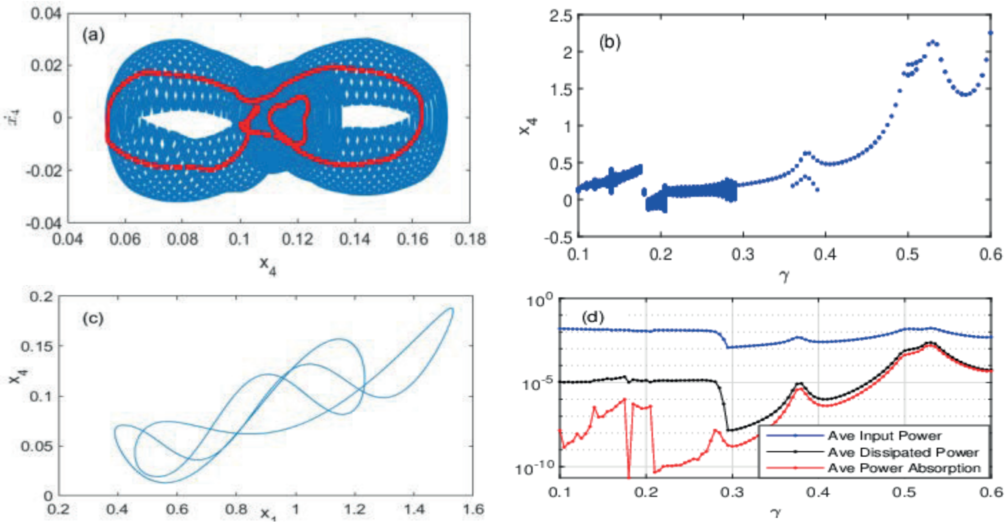

Fig. 2. (a) Phase portrait and Poincare section at $\gamma = 0.22$, (b) Bifurcation diagram of mass m_4, (c) Lissajous curve showing synchronisation of M_1 and m_4, (d) Power plots from energy transfer

Conclusion

Synchronization results from the interactions between the self- and parametric excitations, and are vivid in the periodic windows. Energy exchange between the masses is balanced at those points.

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COMPREHENSIVE STUDY OF GALLOPING ENERGY HARVESTERS

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ABSTRACT

In the work we present the assessment of different variants of galloping energy harvesters (GEH). In recent literature one can find many articles that describe different types of GEHs. These differences are naturally reflected in the operational features of the devices, but so far they were not concluded
in the form of a coherent summary. This problem was previously noticed by the authors of the paper [Bibo et al., 2015], in which 1 degree-of-freedom GEH with nonlinear stiffness of various types are compiled, but due to the very limited scope of the research and not fully analytical results, the study is not exhaustive.

Our study shows that comprehensive information on the performance of GEH of any design can be presented using three parameters: peak efficiency η_p, critical speed u_{cr} and high-efficiency bandwidth B (Fig. 1). Therefore, we provide the method by means of which, these quantities can be formulated analytically and optimized for devices with a galloping resonator, taking into account the impact of non-linear stiffness characteristics in each of the degrees of freedom. The potential benefits of connecting resonators in series will also be explored, which, to the best of author's knowledge, has not yet been done.

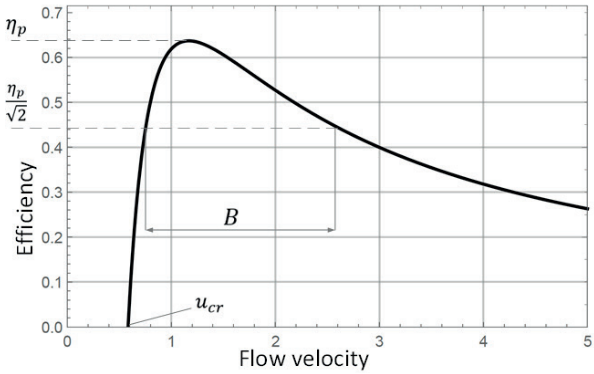

Fig. 1. Efficiency characteristic of galloping energy harvester

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TWO DIMENSIONAL MECHANICAL MODEL OF HUMAN STABILITY IN EXTERNAL FORCE-CAUSED FALL

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ABSTRACT

The first step in any project of medical devices starts with the description of a problem and is usually based on models of movement patterns that are going to be improved. Thereto, it is crucial to acquire knowledge about locomotion, described by well-known mechanical laws. The paper concerns the backward falls caused by a stochastic external force. Although the loss of stability is considered in many literature sources [1,2], the main aim of our studies is to recognize the symptoms of the fall, not to describe how the body works during the fall. The paper involves the description of the methodology of the designed research. The behaviour of a person subjected to a stochastic force is registered by the motion capture system, force plates, and a dynamometer. The synchronization of the signals from these various devices and the correct placement of passive markers on the assessed person's body are prerequisites for obtaining reliable data for further analysis. The acquired data, after preliminary processing, will then be used as input variables in a 2D model of human stability. The model serves among others to calculate moments of forces generated by muscles in joints. To assess the accuracy of the proposed model the output information is compared to data acquired with one of the dynamometric platforms.

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REGISTRATION, MODAL DECOMPOSITION AND ANALYSIS OF HUMAN LEFT VENTRICLES

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ABSTRACT

Cardiovascular diseases, especially myocardial infarction and heart failure, are responsible for a significant proportion of all deaths in the European Union. A proper, timely diagnosis can be a key factor in reducing the mortality of these diseases.

In the present paper, statistical data analysis of the left ventricle of human heart is presented. Raster data, in the form of 3D+t (spatio-temporal) DICOM images from Sunnybrook Cardiac Data collection [1], is processed and registered to obtain geometrical shapes of constant topology. Registered geometrical data, obtained for the whole cardiac cycle of patients with healthy hearts, hypertrophy, and heart failure, is then decomposed using Principal Component Analysis [2], leading to the separation of spatial (modes) and temporal variables (mode coefficients).

This approach is similar to the one proposed earlier in [3], where PCA was used to decomposecardiac cycles for each patient separately, and it was based on the models prepared in [1].

The most of the information about the movement of the left ventricle is covered by the first PCA mode (Fig. 1) - it explains 68.42% of the variance. The second mode explains 7.02% of variance and the first three modes are enough to explain 80.52% of total variance of the input data.

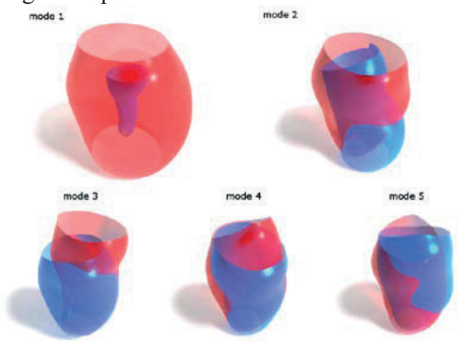

Fig. 1. The five most dominant PCA modes for SCD dataset. The modes are superimposed on the average geometry, multiplied by the maximum (red) and minimum (blue) values of the temporal coefficients

The proposed approach allows neglecting unimportant, noisy signals and the interpretation of the heart cycle data. It is shown that low-dimensional model resulting from modal decomposition might be used to distinguish the hearts with heart failure and the group containing healthy hearts and the ones with hypertrophy, on the base of the averages and the amplitudes of the temporal coefficients of the first PCA mode (Fig. 2).

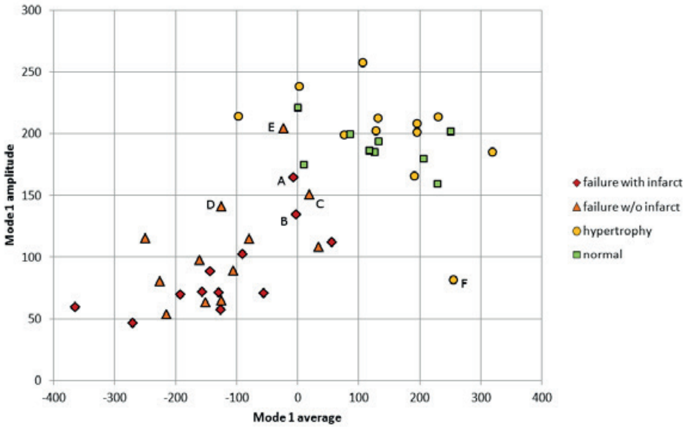

Fig. 2. Cycle-average and amplitude values of temporal coefficients for the first PCA mode. The symbols: diamonds, triangles, circles and squares represent patients with heart failure with and without infarct, with hy- pertrophy and with healthy hearts, respectively.

The paper describes registration process leading to 3D, spatio-temporal surface models and - further - low-dimensional analyses of left ventricles of human heart. Modal decomposition gives more insight into the heart than 2D analysis of MRI slices and can signi
cantly support the diagnosis.

The present work shows that Principal Component Analysis might be used in the classification of the hearts to support the diagnosis process for heart failure. The proposed methodology can detect abnormalities that cannot be observed in "live" diagnostics - before any dysfunctions or symptoms occur. This would help prevent changes in the myocardium through early diagnosis and pharmacological or surgical treatment.

The main meaning of this work is that, being a noninvasive method, it enables the diagnosis of various hearts, including prenatal ones.

Acknowledgments

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DYNAMIC CHARATCERISTICS OF MULTILOBE JOURNAL BEARINGS WITH THE LOBES OF DIFFERENT GEOMETRY

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ABSTRACT

Current status and aim of development of turbounits in achieving higher efficiency and the reduction of costs yields to higher rotor weights and consequently to enlarged specific bearing loads. Finally, the bearings can become the limiting factor for the turbine design and for the efficiency of turbo generator set [1, 2].

Theoretical and experimental investigation into new types of 2-lobe journal bearings of turbo generator [2] points out on the increase in the bearing load capacity. The resistance to motion was reduced by 25%. Such bearing should meet the rotor dynamic criteria, be interchangeable with existing bearings and the friction losses as well as the lube oil supply requirements should be maintained.

New dynamic properties of multilobe journal bearings can be obtained by application in one bearing [2,3] the lobes with a circular, classic multilobe, pericycloid profile. The design of such a bearing allows obtaining new static and dynamic characteristics.

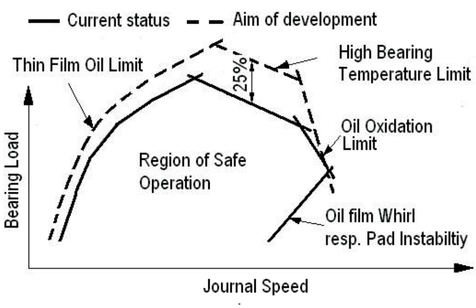

Fig. 1. Current status and aim of development of turbounits journal bearings [2].

The characteristics obtained were used for the determination of the stability of elastic, symmetric rotor. Investigation was carried out at equal or different geometry of bearings lobes.

It was stated that the different geometry of lobes of multilobe journal bearings has an effect on their static and dynamic characteristics and stability of rotor operating in such bearings. This fact gives new possibilities in the investigations into the problems of high-speed bearings.

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CINEMA HALL ADAPTED FOR OPERA SINGING – ACOUSTIC ASSESSMENT

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ABSTRACT

Room acoustics must be adapted to the intended function, and acoustic requirements vary depending on the type of sound production. Acoustic investigations were carried out in the hall of the Olimpia Academic Music Theater (Fig. 1), which was originally a cinema.

Fig. 1. View of the hall from the stage

Acoustic requirements for cinemas are entirely different from those for opera singing [1]. To ensure good acoustic conditions in the interior, an architectural adaptation or introduction of variable acoustics is necessary [2, 3]. The analyzed interior was adapted to the needs of opera singing, and the cinema function was abandoned. The objective of the research is to evaluate the acoustic quality of the hall for the purposes of an opera house. Acoustic tests were carried out using an omnidirectional sound source, DIRAC software together with the Brüel & Kjær ZE-0948 USB sound card. The e-sweep signal was generated and the parameters considered in the literature to be basic for the evaluation of acoustic properties of an opera house were measured. In addition, a pistol shot was used as a sound source.

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STABILITY AND VIBRATIONS OF PLATES IN AXIAL FLUID FLOW

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ABSTRACT

Stability and vibrations of a thin simply supported plate immersed in axial liquid flow on its upper side is studied theoretically. The plate is placed into a hole of an infinite rigid wall (see Figure 1).

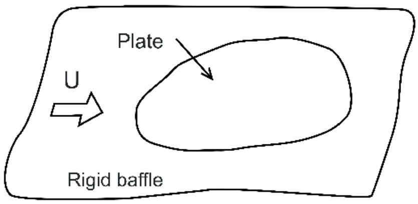

Fig. 1. A plate in flow

The analyzed problem is a coupled problem of the fluid-structure type (see: e.g. [1-3]). It is assumed that the fluid is inviscid, incompressible and the flow is potential. The boundary integral equation is used for describing the hydrodynamic pressure. The plate equation is formulated in the form of two harmonic equations. The problem is solved by numerical methods, such as the finite element method (FEM) and the boundary element method (BEM). The surface of the plate is discretized using triangular curvilinear 6-node elements of the membrane type. These elements are simultaneously the finite elements for the plate and the boundary elements for the liquid. The stability of the coupled system is analyzed by solving the quadratic eigenvalue problem as a function of the flow velocity U. The complex frequencies of the structure enable one to predict whether the structure motion is stable or unstable. The flow velocity at which the first frequency equals zero is the critical velocity of the divergent type (static loss of stability). Numerical examples for circular, square and rectangular plates are considered.

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CHANGE IN THE DYNAMIC PROPERTIES OF A COLUMN AS A RESULT OF A VARIABLE DISTRIBUTION OF BENDING STIFFNESS - NUMERICAL AND EXPERIMENTAL RESEARCH

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ABSTRACT

The subject of the work is the transverse vibrations of a bar system with a variable cross-section. The physical model of the system was subjected to a follower force directed towards the positive pole (Tomski's load) [1, 2]. The variable cross-section was modeled by division of the bar into segments while maintaining the condition of a constant total volume [3]. The increments in the width of individual segments were determined for two functions: linear and 2nd degree polynomial.
The problem was formulated on the basis of the Bernoulli-Euler theory. Then, based on Hamilton's principle, differential equations of motion and boundary conditions were determined. The solution of the equations of motion substituted for the boundary conditions made it possible to develop a computational algorithm for the frequency of vibrations of the system, taking into account variable stiffness, assuming a constant total volume.
On the basis of the obtained results, the relationship between the column stiffness distribution and the eigenvalues of the system was determined. The most favorable and unfavorable outlines of the shape of the column were also determined due to the loss of its stability. The correctness of the obtained results was verified for selected column outlines, indicating the sources of discrepancies. It was shown in the work that by appropriate shaping of the bar it is possible to control its dynamic properties.

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FREE VIBRATIONS OF A FLAT FRAME PARTIALLY RESTING ON A WINKLER ELASTIC FOUNDATION IN TERMS OF UNEVEN DISTRIBUTION OF FLEXURAL STIFFNESS

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ABSTRACT

The work was devoted to the issue of the free vibrations and loss of stability of a gamma-type flat frame made of a prismatic beam and a column with a variable cross-section [1], partially supported on a Winkler elastic foundation [2,3]. The physical model of the system was subjected to the Euler's force. The study analyzed various mounting variants.

The problem was formulated on the basis of the Bernoulli-Euler theory. Based on the principle of minimum potential energy and Hamilton's principle, differential equations of displacements and boundary conditions were determined. The solution of differential equations substituted for the boundary conditions made it possible to develop an algorithm for the calculation of the critical force of the system, taking into account the variable stiffness and different positions of the Winkler support.

On the basis of the obtained test results, the relationship between the column stiffness distribution, the location and length of the support by the elastic foundation and the free vibrations as well as critical forces of the system was determined. The ranges of the tested parameters were determined for which the increase in the buckling load as compared to the unsupported prismatic system was the greatest.

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EXPERIMENTAL DYNAMICS ANALYSIS OF THE THREE-BLADED ROTOR

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ABSTRACT

The experimental studies are important part of dynamics analysis, which allow to compare advanced numerical and analytical results with reality. This experimental contribution is divided on two parts, firstly investigates the natural frequency of the three-bladed rotor and next part is focused on the analysis of aerodynamics loads of the rotor. The test stand was designed for scientific researches and experiments with constant velocity or excitation of any optional signal, as well as excitation with different frequencies.

The laboratory stand is presented in Fig. 1 and is composed of the rotating hub and three slender composite beams. The beams are made of a graphite-epoxy prepreg material. Each blade has a length of 595 mm, the cross section 34×0.9 mm. Near the fixed end of each beam, the strain gauge is embedded. In order to measure strains not only in the longitudinal direction, which capture elongation and flap-wise bending deformations, but also indirect chord-wise deformations and torsion a rosette strain gauge. The sensor is positioned along the span of the beam in its axis, and two measuring grids in $\pm 60^{\circ}$ from the main axis of the beam. The signals from the strain gauges are sent to computer software by the wireless module. The radius of the hub is 120 mm. In the hub there are boards with electronic circuits with the wireless module, which allows to transmit signals from the beams. The laboratory set-up is excited by the DC motor with power of 1.1 kW (1.5 HP).

Fig. 1. Rotor with three slender beams — laboratory stand

Firstly, the natural frequency of cantilever beam has been obtained by the non-contact scanning laser vibrometer Polytec PSV-500. Then, the forced vibrations of the rotor have been analyzed. The frequency of excitation has been changed from 1.0 Hz to 22 Hz with step 0.1. The laboratory stand has been driven for each value of the excitation frequency at least one minute. Next, based on the signals from the strain gauges time series have been prepared. The frequency response curves have been drown based on the amplitudes, which have been interpreted from the time histories. Finally, results obtained from experimental tests are compared with numerical simulations from the finite element method [1].

The second part of the experimental studies has been focused on the aerodynamical loads of three- bladed rotor. During the laboratory tests the angular speed of the hub, as well as the preset angle have been changed. For each angular velocity (50, 100 and 150 rpm) four different preset angles have been set (0°, 5°, 45° and 90°). For more precision, two types of measurements have been used - two high-speed cameras and set of strain gauges. The cameras have been used for optical contactless mea- surements, while strain gauges embedded on each beam allowed to record time series during the beam deformations and rotation of the system. Additionally, based on time series the synchronization phe- nomenon is analyzed [2].

In future work, the collected results will be used to adopt previously elaborated theoretical results. Furthermore, an attempt will be made to introduce proportional and cubic type control algorithms as well as combined proportional-cubic control algorithms with the use MFC patches [3].

Acknowledgments
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STRUCTURAL HYBRID MODELLING APPLIED TO INVESTIGATE CURRENT PROBLEMS OF ROTOR DYNAMICS

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ABSTRACT

The notion of rotor dynamics should be defined as a branch of dynamics of mechanical systems including dynamic investigations of machines and devices, the main working elements of which are under rotational motion. One of the main subjects of the rotor dynamics is a quantitative and qualitative examination of steady-state and transient bending/lateral, torsional and axial vibrations of these elements and the influence of these vibrations on the environment and cooperating objects, e.g. electric motors and generators. Because of design and technological reasons, these fundamental rotating operational elements are usually in the form of stepped rotor-shafts with segmentally constant or approximately constant cross sections. Such relatively simple geometric shape and realistic frequency ranges of practically tested vibration processes allow the use of one-dimensional structural dynamic models of the beam type ensuring an appropriate efficiency and reliability of results of theoretical analyzes carried out for this type of mechanical systems. According to the above, a one-dimensional discrete-continuous structural modeling seems to be a particularly useful modeling method for studying dynamic processes in rotating machinery. It consists in a hybrid combination of flexurally, torsionally and longitudinally deformable continuous beam finite macro-elements representing individual cylindrical or quasi-cylindrical segments of the real rotor-shaft with discrete oscillators corresponding to its bearing supports, rotor disks, coupling flanges, flywheels, etc., in accordance with a structure of the real object. Mutual connections of the successive macro-elements creating the stepped shaft as well as their interactions with the discrete oscillators and other objects, e.g. electric machines or magnetic supports, are described by equations of boundary conditions. These equations contain geometrical conditions of conformity for translational, rotational and axial displacements of extreme cross sections of the adjacent continuous elastic macro-elements. The second group of boundary conditions are dynamic ones, which contain linear, nonlinear and parametric equations of equilibrium for external forces and torques, static and dynamic unbalance forces and moments, inertial, elastic and external damping forces, support reactions and gyroscopic moments. For the local partial differential equations of motion of these macro-elements there are applied analytical solutions in the form of series in orthogonal eigenfunctions leading to the description of motion in modal coordinates. Such approach allows, on the one hand, to effectively obtain simulation results in the time domain of linear and non-linear forced vibrations and quickly-varying shock processes. On the other hand, they make it possible to conduct qualitative tests of the considered rotor-shaft systems in the form of natural vibration analyzes, generation of amplitude-frequency characteristics and determination of dynamic stability limits. Moreover, a relatively clear mathematical description of the adopted discrete-continuous (hybrid) models of the rotor-shaft systems enables us a convenient coupling of motion equations of the mechanical objects under study with circuit voltage equations being mathematical models of electric motors and generators cooperating with these objects, or with equations describing an interaction of magnetic supports.

Summing up, it should be stated that the degree of complexity of the mathematical description and numerical advancement necessary to obtain reliable solutions using this method of modeling of the

above-mentioned objects can be classified as a kind of intermediate between the classical finite element modeling and traditional discrete modeling, now called "multi-body modeling". Owing to this, an application of the proposed structural discrete-continuous (hybrid) models for dynamic analyzes is associated with much smaller problems of physical parameters identification compared to a usually burdensome determination of such parameters of discrete models of respectively the same objects. As a result, this leads to an achievement of actual accuracy of calculations results comparable or identical to those obtained using analogous FEM models, while additionally there is obtained much greater numerical efficiency justified by the need to solve smaller numbers of mathematical relationships describing motion of discrete-continuous (hybrid) models.

The following examples will illustrate the calculation possibilities of the presented methodology:

1. Coupled lateral-torsional-axial vibrations of the steam turbogenerator shaft-line with a transverse crack. Here, as described in [1,2], the torsional and axial vibrations induced by the local cracked shaft anisotropy will serve as diagnostic symptoms for identification of this kind of fault.
2. Stability analysis of the automotive turbocharger rotor supported by the electrodynamic passive magnetic bearings. In order to stabilize such rotating system several ways of external damping introduction into the rotor-shaft suspension will be proposed, [3,4].
3. Dynamic investigations of the industrial fan with imperfections caused by static and dynamic unbalances of its heavy overhung rotor, parallel and angular misalignments of its drive shaft segments, inner anisotropy of couplings connecting its successive rotor-shaft segments and faulty stagger angles of its rotor-blades responsible for abnormal pressure pulsation exciting additional lateral vibration components. An influence of the above listed imperfections on sensitivity and stability of such rotor-shaft system will be tested, as demonstrated in [5].
4. Transient self-excited torsional vibrations of the electric locomotive drive system induced by a frictional interaction between the running wheels and the rails for various traction conditions.
5. A suppression of steady-state and transient torsional vibrations of the rotor machine driven by an asynchronous motor serving as a simultaneous source of drive and actuator. Here, severe resonances and rapid dynamic overloadings will be effectively attenuated by means of active motor control methods properly modified in comparison with standard ones described in [6].

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AMPLITUDE-DEPENDENT EFFECTS OF ULTRASOUND IN BUBBLY LIQUIDS

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ABSTRACT

The presence of tiny gas bubbles in a liquid, even in a small proportion, changes its acoustic properties drastically. These changes have huge effects on the propagation of ultrasound in bubbly liquids. The highly nonlinear and dispersive characters of the medium allow the observation of some special features, such as the generation of new frequency components from a monochromatic ultrasonic wave of finite amplitude propagating through the bubbly liquid. The work presented here focuses on several nonlinear aspects of these phenomena: the generation of harmonics and subharmonics in resonant cavities, the softening of the medium, and the optimization of the bubbly liquid. To this purpose, we use a numerical model developed previously, which solves a system of two coupled differential equations, the wave equation and a Rayleigh-Plesset equation.

Acknowledgments
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KINETIC ENERGY BASED INDICATORS TO COMPARE DIFFERENT LOAD MODELS OF A MOBILE CRANE

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ABSTRACT

Three different variants of mathematical models of a load carried by a mobile crane are examined and compared in the paper. The considered mobile crane is the form of a tree structure of a kinematic chain with auxiliary chains and a carried load is modelled in three variants: as a lumped mass on one hook sling (3-dof [1]), as a sphere on one hook sling (6-dof) and as a box on four hooks sling (6-dof [2]) – Fig.1. The crane input movement is divided into five phases – Fig.2.

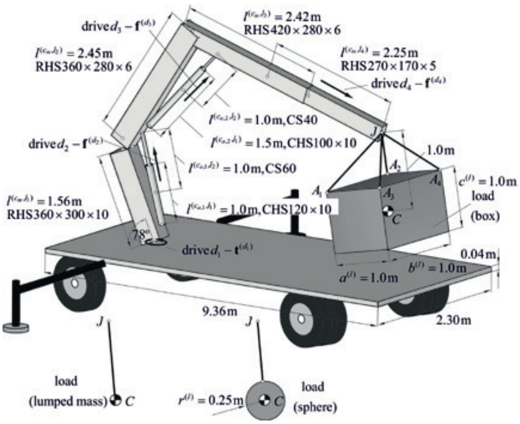

Fig. 1. Model of the crane and the load

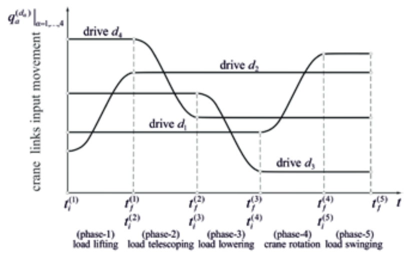

Fig. 2 Input movement phases of the crane links

Vector of the generalized coordinates is defined as follows

$$\mathbf{q} = \left[\mathbf{q}^{(c)^T} \quad \mathbf{q}^{(l)^T} \right]^T , \tag{1}$$

where: $\mathbf{q}^{(c)}$, $\mathbf{q}^{(l)}$ are vectors of generalized coordinates of the crane and load, respectively.
The influence of load modeling methods on the kinetic energy of the load is estimated using following indicators

– integral mean value

$$\left.\bar{E}_k^{(l)}\right|_{t\in\left[t_i^{(\alpha)},t_f^{(\alpha)}\right]} = \frac{1}{t_f^{(\alpha)}-t_i^{(\alpha)}} \int_{t_i^{(\alpha)}}^{t_f^{(\alpha)}} E_k^{(l)}(t)\mathrm{d}t \, , \tag{2.1}$$

– arithmetic average deviation from the integral mean value

$$\hat{E}_k^{(l)}\Big|_{t\in\left[t_i^{(\alpha)},t_f^{(\alpha)}\right]} = \frac{1}{n}\sum_{j=1}^{n}\left|E_k^{(l)}(t_j) - \bar{E}_k^{(l)}\right|. \tag{2.2}$$

The time courses of kinetic energy of the three variants of the carried load are shown in Fig. 3. Figs 4 and 5 present the values of the indicators defined by Eqs 2.

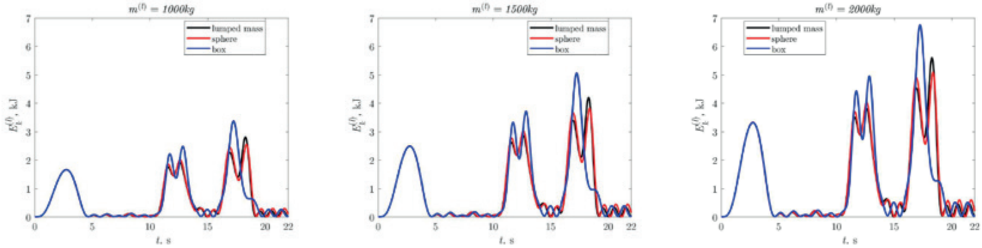

Fig. 3. Time courses of the kinetic energy of the load

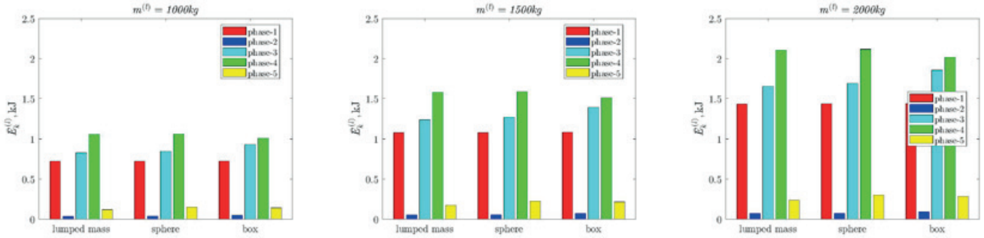

Fig. 4. Integral mean value of the kinetic energy of the load in phases of input movement

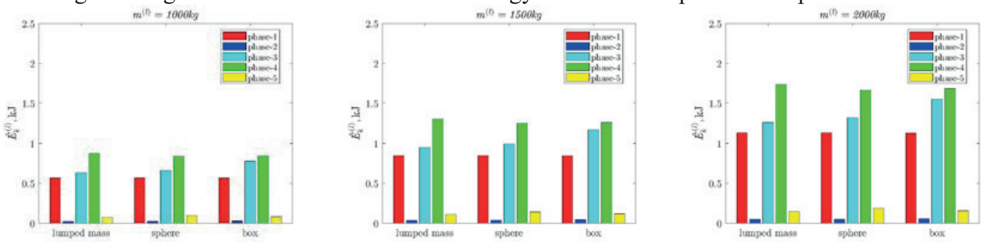

Fig. 5. Arithmetic average deviation from the integral mean value of the kinetic energy of the load in phases of input movement

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LOAD ANALYSYS OF HYDROFOIL WINDSURFING ATHLETE

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ABSTRACT

Foil windsurfing is a relatively new sport discipline and from the point of view of both the swimming technique and the positioning of the athletes on the board, there are still many unknown elements. While the optimal position of the surfer is still being sought, an important issue is what values are achieved by the loads (forces and moments) in the joints of the person swimming with such equipment. The paper presents a two-dimensional mechanical model of the athlete, which will be used to determine the loads on individual segments of his body depending on the position on the board. The three positions most often taken by athletes on the board while swimming will be examined. The motion capture system: BTS Smart will be used to accurately measure key points of the body. The necessary geometric and mass data of each of the tested athletes will come from anthropometric measurements, the results of which will be applied to the extended Clauser model. The results should answer the questions about which position is the most favorable from a biomechanical point of view, which joints are most loaded, and which part of the load is taken by the line connecting the equipment with the athlete.

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GAIT SYMMETRY ASSESSMENT AFTER TWO-SIDE LOWER LIMB AMPUTATION

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ABSTRACT

The aim of the study is to investigate the gait symmetry of various prosthetic solutions in a person after two-side lower limb amputation. The patient was familiar with all types of prostheses tested and moved with their help efficiently. The study was carried out using the BTS SMART motion capture system together with dynamometric platforms. Standard, a set of parameters known as gait determinants are used to biomechanically assess human gait. They allow for the assessment of the correctness of gait and the indication of abnormalities and compensation mechanisms in the gait of the examined person. Their values are also easy to compare with numerous examples of literature data. However, they do not fully describe the asymmetry between the sides of the body, which is particularly important in the analysis of gait in prosthetic appliances. Therefore, the paper proposes a set of parameters that leave the difference in kinematic and dynamic quantities, such as components of the ground reaction forces oriented at determining the asymmetry. The focus was primarily on assessing the shape of key characteristics, not just values. Such an analysis will allow not only to assess the correctness of gait, but also to assess the calibration of orthopedic equipment and the selection of the optimal solution for the patient under study.

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DYNAMIC ANALYSIS OF HIGH COOLING TOWERS

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ABSTRACT

This study is to determine and to compare the dynamic characteristics of cooling towers for various variants depending on the support and stiffening of the upper edge of hyperboloid and catenoid shells.
Computer models of the analysed structures were loaded with a mining tremor and the stresses and displacements were caused by this rock bump were calculated.
The structure that was analysed by the authors and based on which alternative cooling towers were created was a 186 [m] hyperboloid cooling tower located in Opole.
Common features of all analysed cases are:

1. narrowing of the cooling tower,
2. connection point of the columns supporting the shell with the foundation.

These features are marked in the figure below.

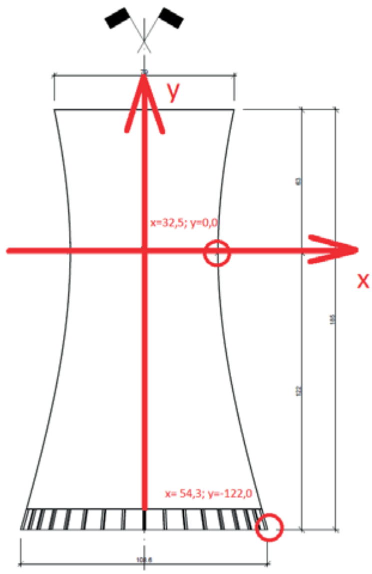

Fig. 1. Common features of all analysed cooling towers

The calculations were carried out using the finite element method in Robot Structural Analysis Professional. Now the calculations are run in ANSYS program.
The results of the calculations show that catenoid cooling towers are stiffer than hyperboloidal cooling towers, and therefore they experience greater stresses with the same acting force.
The way of supporting of the shell has the biggest influence on dynamic characteristics and resultant stresses during the seismic or para seismic forces.

NON-LINEAR DYNAMIC RESPONSE OF A GUY LINE OF A GUYED TOWER TO THE STOCHASTIC WIND EXCITATION BY EQUIVALENT LINEARIZATION TECHNIQUE

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ABSTRACT

Non-linear behaviour of the guy line of a guyed tower idealized as an isolated taut string is considered. The guy line is subjected to the stochastic base motion excitation which is the displacement of the point of attachment of the guy line to the tower resulting from transverse vibrations of the tower due to wind. Under strong wind excitation the transverse displacements of the tower may be large, which causes the geometrical non-linear effects. Therefore the partial differential equations governing the coupled axial and transverse non-linear vibrations of the string are assumed [1]. The Galerkin method is used to convert a problem into the one governed by ordinary differential equations. Under the assumption that the tower structure responds essentially in the single (fundamental) mode, the response of a tower to a wide-band stochastic wind excitation is a narrow-band stochastic process. Consequently the base motion excitation for the string vibrations is a narrow-band stochastic process. This process is modelled as a response of an auxiliary linear filter to a Gaussian white noise excitation [2]. As a result, the state vector of the system consisting of original state variables augmented by state variables of a filter is driven by Gaussian white noise and is a diffusive Markov process. The equivalent linearization technique is applied, where the original non-linear system is replaced with a linear one, whose coefficients are determined from the condition of the minimization of the mean square error between the original non-linear and the equivalent linear system. The mean value and variance of the response are determined and those approximate analytical results are verified against direct numerical Monte Carlo simulations.

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A DOUBLE PENDULUM WITH FRICTION UNDER THE ELECTROMAGNETIC FORCING AND KINEMATIC EXCITATION: SIMULATION AND EXPERIMENTATION

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ABSTRACT

A mathematical model of a double pendulum with friction is derived from the extended swinging Atwood machine. The novel system is modeled under electromagnetic forcing and kinematic excitation. Furthermore, we present some results from the computational analysis and the system's real-time laboratory set-up for experimentation. The system shows promising results as more observations based on system identifications are in progress.

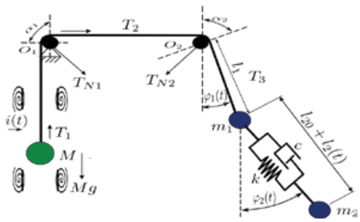

Fig. 1. Schematic diagram of the proposed double pendulum under the influence of electromagnetic forcing

The system is modeled with a frictional force acting on the two pulleys under the action of electromagnetic forcing. The equation of motion is derived from Newton's second law of motion and verified by the Euler-Lagrange method. The friction has to be modeled as well for better apprehension and precise responses to the dynamical system by using extra state variable T_R Eqs. (1)-(4).

Equations of Motion

$$\ddot{l}_1(t) = \frac{-(m_1 s_{f2}+M)\ddot{X}_0+m_1(l_1(t)\dot{\varphi}_1^2+gc_{f1})+F_{ck}(c_{f1}c_{f2}-s_{f1}s_{f2})+T_R-Mg}{m_1+M} \tag{1}$$

$$\ddot{l}_2(t) = \left(\frac{1}{m_1 m_2(m_1+M)}\right)\Big(-\Big(\big(s_{f2}(s_{f1}^2-s_{f1})+c_{f1}c_{f2}s_{f1}-c_{f1}c_{f2}\big)Mm_1m_2\ddot{X}_0+Mm_2s_{f1}^2F_{ck}(1-2s_{f1}^2)+Mm_2\big(m_1l_1(t)\dot{\varphi}_1^2(t)+2c_{f1}c_{f2}F_{ck}+gm_1c_{f1}\big)+m_1m_2s_{f1}s_{f2}(T_R-Mg)+Mm_2s_{f1}^2\big(F_{ck}+gm_1c_{f2}\big)-\big(m_1m_2(m_1+M)(l_2(t)+l_{20s})\big)\dot{\varphi}_2^2(t)-Mm_1m_2c_{f1}c_{f2}l_1(t)\dot{\varphi}_1^2(t)+(m_2+m_1+M)m_1F_{ck}+m_1m_2c_{f2}\big(c_{f1}T_R-Mgc_{f1}-Mg\big)\Big)\Big) \tag{2}$$

$$\ddot{\varphi}_1(t) = -\frac{m_1(2\dot{l}_1(t)\dot{\varphi}_1(t)+c_{f1}\ddot{X}_0+gs_{f1})+F_{ck}(s_{f1}c_{f2}-s_{f2}c_{f1})}{m_1l_1(t)} \tag{3}$$

$$\ddot{\varphi}_2(t) = \left(\frac{1}{(m_1+M)(m_1l_2(t)+m_1l_{20s})}\right)\Big(\Big(\big((s_{f1}-1)s_{f2}c_{f1}-s_{f1}c_{f2}(s_{f1}-1)\big)Mm_1\ddot{X}_0-2Ms_{f1}c_{f1}s_{f2}^2F_{ck}+\Big(\big(2c_{f2}s_{f1}^2F_{ck}+gm_1s_{f1}^2-m_1c_{f1}l_2(t)\dot{\varphi}_1^2(t)-c_{f2}F_{ck}\big)M+(T_R-Mg)m_1c_{f1}-Mgm_1\Big)s_{f1}+\Big(\big(m_1c_{f2}l_1(t)\dot{\varphi}_1^2(t)+c_{f1}F_{ck}\big)M+\big(Mgc_{f1}-T_R+Mg\big)m_1c_{f2}\Big)s_{f1}2m_1\dot{l}_2\dot{\varphi}_1^2(m_1M)\Big). \tag{4}$$

The friction variable T_R is the sum of the friction at the first pulley T_{R1} and the friction at the second pulley T_{R2}. That is $T_R = T_{R1} + T_{R2}$ where $T_{R1} = \mu_{p1} T_{N1} + T_{pv1}(\dot{l}_1)$, $T_{R2} = \mu_{p2} T_{N2} + T_{pv2}(\dot{l}_1)$, $T_{N1} = \sqrt{T_1^2 - 2T_1T_2\cos(\alpha_1) + T_2^2}$, $T_{N2} = \sqrt{T_2^2 - 2T_2T_3\cos(\alpha_2) + T_3^2}$, $T_1 = Mg - m_1\ddot{l}_1(t)$,
$T_2 = \frac{T_1\left(2+sin(\dot{l}_1)\mu_{p1}\sqrt{2(1-\cos(\alpha_1))}\right)+2T_{pv1}(\dot{l}_1)}{\left(2-sin(\dot{l}_1)\mu_{p1}\sqrt{2(1-\cos(\alpha_1))}\right)}$, $T_3 = \frac{T_2\left(2+sin(\dot{l}_1)\mu_{p2}\sqrt{2(1-\cos(\alpha_2))}\right)+2T_{pv2}(\dot{l}_1)}{\left(2-sin(\dot{l}_1)\mu_{p2}\sqrt{2(1-\cos(T_R))}\right)}$. The wrapping angle at the first and second pulley is given by $\alpha_1 = 90^0$ and $\alpha_2 = (90 - \varphi_1)^0$ respectively.

Simulation Results and Discussion

The mathematical model of the double pendulum with friction has been solved numerically using the Runge-Kutta method with adaptive step size. The results described the double pendulum with friction concepts demonstrating the system behavior with a suspension system between the two pendulums excited by an electromagnet and armature. The excitation frequency has an impact on the system.

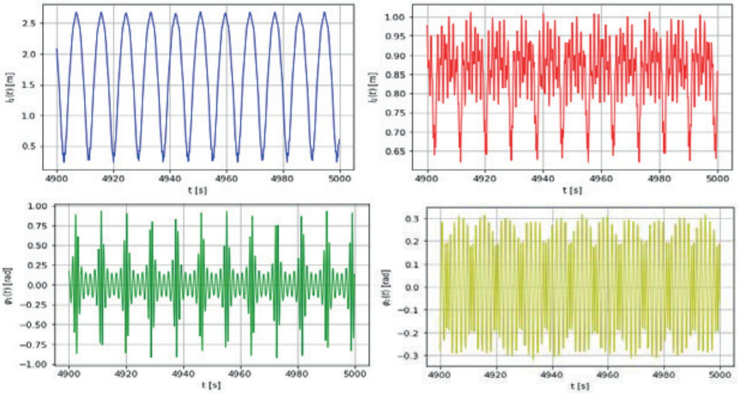

Fig. 2. Numerical results of simulation of the double pendulum: Time Histories for $\ddot{l}_1(t)$, $\ddot{l}_2(t)$, $\ddot{\varphi}_1(t)$, $\ddot{\varphi}_2(t)$

Conclusion

The presented results show the time series of the double pendulum, with no physical contact between the swinging assemble and the fixed points. Most importantly, in some regimes, compact regions of attraction, such as in Fig. 2, appear in the system. Therefore, the nonlinear dynamics of the double pendulum with friction can be vigorously studied, and more possibilities for modification.

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