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All published papers received a positive opinion of the members of the Scientific Committees.

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PLENARY LECTURES

Speaker Biographies

Prof. Antonina Pirrotta is a Full Professor in Structural Mechanics at the University of Palermo, Italy. She graduated in Civil Engineering from Palermo University in 1987. Firstly, for 6 years she worked as freelance structural engineer, then has done PhD in Structural Engineering in 1996 and as Post–doctoral studies in 1998. In 2000, she became a Researcher, in 2001 Associate Professor and in 2016 Full Professor in Engineering department at the University of Palermo, Italy.
-HEAD of PhD School of University of Palermo from 2023.
-HEAD of CIDIS from 2021.
-Board of Governors of ASCE (EMI) 2021-2024 (exceptionally renewed every year).
Her achievements resulted in many scientific recognitions and awards, which include the appointment as Chair of IDEA Innovative Dynamics Experiments Association (2022), several fellowships, memberships on scientific committees of various key international conferences, a membership on the Advisory Board of the MSCA COFUND Doctoral Programme at the University of Innsbruck, and many invitations as plenary speaker at several international conferences.
Prof. Pirrotta has organized and co-organized an exceptionally large number of international conferences, the last EMI2023International Conference organized at University of Palermo and for what she received the appreciation award, minisymposia, and summer schools in Italy and all over the world. She is an Associate Editor of the Journal of Engineering Mechanics (ASCE), (first professor with Italian affiliation) and of Meccanica, which are two absolute top journals in the field. Additionally, she is a member of the Editorial Board of various international scientific journals. She was a Visiting Professor at many universities all over the world such as Tongji University, Columbia University, Liverpool University, TU Wien, etc. With over 180 published papers, she is a leading researcher in the field of structural mechanics and collaborates with renowned institutions worldwide.
Prof. Antonina Pirrotta ‘s contributions and leadership have made her a notable figure in Structural Mechanics and a role model for women in science.

Prof. Piotr Fedeliński was graduated from the Faculty of Mechanical Engineering at the Silesian University of Technology, Poland in 1981. He obtained his Ph. D. degree in 1991 and the D. Sc. degree in 2000. In 2019, he was given the title of professor of technical sciences. He is employed as a full professor at the Faculty of Mechanical Engineering at the Silesian University of Technology. He was Vice-Dean for Science during the 2008–2012 term, and from 2012 to 2016, Vice-Dean for Science and Industry Cooperation. He was the head of the Section of Computational Mechanics and Applied Computer Science. He is the author or co-author of approximately 170 publications, including four monographs. He was the leader of five research projects funded by the Committee for Scientific Research and the National Science Centre, Poland. He was the supervisor of four distinguished Ph. D. theses. He participated in three long-term foreign projects: at Aalborg University, Denmark, in 1991; at the University of Portsmouth, United Kingdom, from 1992 to 1995; and at the University of Campinas, Brazil, in 1999. He has been a member of the Committee of Mechanics of the Polish Academy of Sciences since 2020, a member of the Polish Society of Theoretical and Applied Mechanics since 1995, and a member of the Polish Association for Computational Mechanics since 1997. Professor Piotr Fedeliński carries out scientific research in the fields of computational mechanics, the dynamics of deformable structures, fracture mechanics, micromechanics, sensitivity analysis, and structural optimization. His main area of research is the development and application of the boundary element method (BEM) in solid mechanics. Some of his original achievements include the optimization of shapes of structural elements with respect to criteria dependent on natural frequencies, a comparison of the effectiveness of various formulations of the method in the analysis of dynamically loaded bodies with cracks, modeling of rapid crack propagation, homogenization of cracked materials and composites, and analysis and optimization of dynamically loaded composites. Title of plenary lecture: Boundary element method in dynamics of deformable solids.

Prof. Jarosław Latalski
The fields of scientific research include the dynamics of continuous structures, reduced-order modeling, nonlinear vibrations and oscillations, bifurcations, and nonlinear control of structural systems. Furthermore, research focuses on experimental dynamics, intelligent structures, and energy harvesting. Active materials analysis, composite materials mechanics, and structural optimization are also included in the thematic scope of research.

Prof. Przemysław Litewka was born in Poznań. After graduating in 1992, he began working at the Poznań University of Technology. In 1998, he defended his doctoral dissertation and obtained his habilitation in 2008.
In 2013, he was appointed professor at the Poznań University of Technology. In 2021, he was awarded the title of professor of engineering and technical sciences.
From 1999 to 2001, he was a Humboldt Stiftung scholar at the University of Hannover.
His research specializations include broadly defined structural mechanics, in particular: contact mechanics, the dynamics of structures with viscoelastic materials, and the application of numerical methods to the mechanics of rod systems and surface girders. He is the author of over 100 scientific papers and a member of the Polish Society of Theoretical and Applied Mechanics and the Polish Society for Computational Mechanics.

Abstracts

NEW ADVANCES IN PASSIVE STRUCTURAL VIBRATION CONTROL

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ABSTRACT

Since their first appearance in 1989 in the work by Sakai et al. [1], Tuned Liquid Column Damper (TLCD) devices received growing attention among researchers who deal with structural control. TLCD is in fact of great interest among passive vibration control systems, because of its characteristics such as easy implementation, low cost of construction and maintenance, no need to add mass to the structure if the liquid is used as water supply, and control performance comparable to other passive control devices. TLCDs dissipate structural vibrations through a combined action which involves the motion of the liquid mass within the tube. The restoring force is produced by the force of gravity acting on the liquid, while the damping effect is generated by the hydrodynamic head losses that arise during the motion of the liquid inside the TLCD. If further dissipation is needed, orifices are included inside the vessel.
Although the increasing use of these devices for structural vibration control, it is shown that existing models do not always lead to accurate prediction of the liquid motion.
In this talk, a different formulation of the equation of motion of the TLCD liquid displacement is presented, taking advantage of the fractional derivative and its properties. Experimental validation of the predicted behaviour is fully developed in frequency and time domain as well.
Moreover, due to the inherent long-period nature of TLCDs, early investigations focused mainly on the application of these devices in flexible structures such as tall and slender structures, which are characterized by high natural periods and are highly prone to wind-induced vibrations, However, seismic excitations encompass a broader frequency bandwidth, rich in high-frequency content compared to wind-induced vibrations, which can be potentially hazardous to both flexible and stiff structures. Tuning the TLCD to match the shorter periods of such structures presents challenges in achieving feasible lengths of the TLCD column tube. The need to tune the TLCD frequency to match the frequency of the structure further limits its application to stiffer structural configurations.
In addition, in this talk it will be introduced how to extend their utility to short-period structures.

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BOUNDARY ELEMENT METHOD IN DYNAMICS OF DEFORMABLE SOLIDS

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ABSTRACT

Three fundamental formulations of the boundary element method (BEM) used in dynamics of deformable solids will be presented, i.e. the time domain method, the integral transform method and the dual reciprocity method [1]. The advantages and disadvantages of each approach will be discussed. The BEM allows analysis of linear elastic structures subjected to dynamic tractions only by the division of boundaries into boundary elements [2]. Because in this case, the displacements, tractions, and coordinates of points are interpolated along the boundaries, the results are more accurate than in the domain methods, for instance, in the finite element method (FEM). The modification of the geometry is much simpler than in the domain methods, which is very important in problems like shape optimization, identification of defects, modeling of crack growth, etc.
The work will present formulations and applications of the BEM for particular cases of dynamically loaded linear elastic plates. The BEM will be used for modeling of fast growing cracks [3]. The loading of the specimens are taken from experiments. The computer code allows computation of crack shapes which are compared with the experimental results. The branched cracks will be analyzed [4]. The influence of the crack geometry and the interaction between cracks [5] on stresses will be studied. Composites reinforced with rigid straight fibers subjected to harmonically excited vibrations and impact loads will be analyzed [6]. The influence of the length of fibers and their distribution on the displacements will be studied. Finally, the interaction between fibers and cracks in the composites will be investigated. For plates with cracks, stresses at the crack tips are characterized by dynamic stress intensity factors (DSIF), which are calculated using the crack opening displacements. The dynamic SIF are compared with static values. The results are verified by comparison with the analytical or numerical results obtained by the FEM.

Acknowledgments
Scientific research is financed by the statutory subsidy of the Faculty of Mechanical Engineering of the Silesian University of Technology in 2026.

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NOLINEAR DYNAMICS OF ACTIVE BEAM-LIKE STRUCTURES

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ABSTRACT

Over the past decade, there has been a growing interest in the application of active materials in structural design, including aerospace, mechanical, and civil engineering applications. Among active materials, piezocomposites are the most widely used, in particular as micro-fibre patches (MFC).

In structural modeling, piezoelectric composite materials are typically assumed to be homogeneous, with effective electroelastic properties determined by their constituent materials. Linear constitutive models, which link elastic and dielectric characteristics, are commonly used [1]. These models are sufficiently accurate under low to moderate mechanical loads or voltage levels assumption. However, more advanced models capture nonlinear effects associated with large displacements, strains, or nonlinear constitutive relations [2,3].

This research aims to experimentally study the electromechanical properties of piezocomposite patches in the nonlinear regime. Based on laboratory results, a novel, validated mathematical framework for modelling active structures is developed.

In the experiments an MFC bimorph cantilever was tested subject to kinematic excitations. The tests were conducted over a broad range of mechanical and electrical operating conditions near the fundamental resonance zone. The frequency responses in mechanical and electrical domains were recorded, as well as corresponding phase shifts. The results revealed a softening-like dynamic behavior of a structure with a non-classical, unique linear backbone curve.

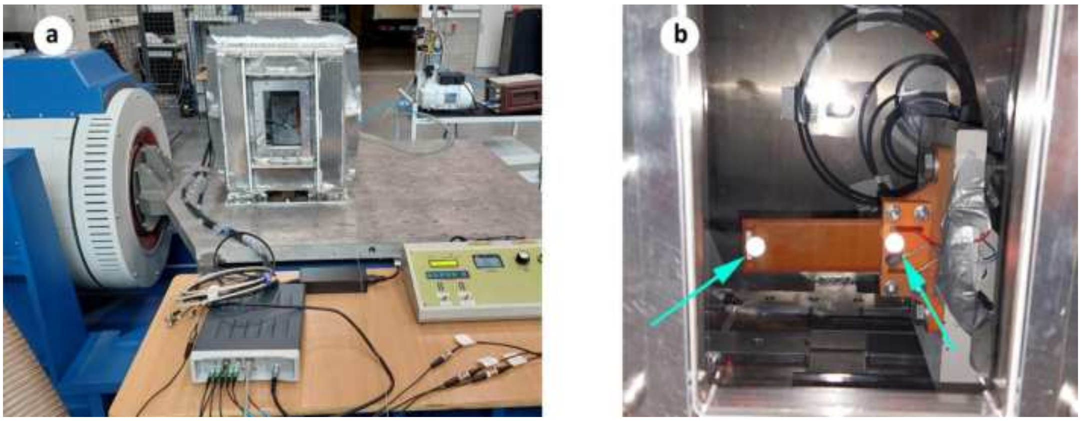

Figure 1: Laboratory test stand: (a) vacuum chamber on slip table driven by shaker,
(b) tested MFC patch in cantilever bimorph configuration (b).

Next, an analytical model of the tested bimorph was proposed and derived based on the Hamilton's principle. It is represented by a system of two coupled ODEs in the time domain

$$\ddot{Q} + \alpha_{11}\,\dot{Q} + \alpha_{12}\,Q + \alpha_{13}\,Q^2\,\mathrm{sgn}(Q) + \alpha_{14}\,Q^3 + \alpha_{15}\,Q\,V\,\mathrm{sgn}(Q) = \alpha_{16}\cos(\Omega t)$$

$$\alpha_{21}\,\dot{V} + \alpha_{22}\,V + \alpha_{23}\dot{Q} + \alpha_{24}Q\,\dot{Q}\,\mathrm{sgn}(Q) = 0$$

addressing the mechanical and electrical domain dynamics, given by Q and V variables respectively.
Next, the harmonic balance method was used to derive the adjoint system of equations in magnitudes of system responses and their phase shift ϕ_{EM}. These equations, together with the recorded frequency response curves, were used to identify the unknown material nonlinear properties of the piezocomposite MFC patch.
Finally, the structural model of a rotating hub and clamped bimorph beam carrying a tip mass was studied analytically and numerically. In the proposed formulation the identified nonlinear properties of the MFC patch were adopted. The dynamics of the studied system is represented by a system of three coupled nonlinear integro-partial differential equations capturing the electro-mechanical behavior of the beam (transverse displacement and transducer output voltage) and the angular coordinate of the hub. Next, the derived governing equations were reduced by virtue of the Galerkin method and solved numerically around the first resonance zone under periodic torque excitation supplied to the hub. The performed numerical simulations showed the system performance for different scenarios of torque excitation, tip mass ratios and electrical boundary conditions.

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DYNAMICS OF PLATES WITH VISCOELASTIC LAYERS DESCRIBED BY EXTENDED FRACTIONAL MATERIAL MODELS

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ABSTRACT

Laminate plates are frequently used structural elements in many branches of engineering. Understanding of their dynamic behaviour under various service conditions is of great importance. This lecture is devoted to the free vibrations and non-linear harmonically excited vibrations of such plates containing viscoelastic layers, which are useful in reducing excessive vibrations. The plate kinematics is described with the refined zig-zag theory. The viscoelasticity is modelled using a generalised scheme valid for four different material descriptions based on the fractional derivatives. Discussion of the models performance in the representation of experimental results will be presented. The time-temperature superposition and three methods of thermal shift computation are considered to include the effect of the ambient temperature. The lecture will present results and discussion of several numerical examples. In particular, comparison of the damping performance of six different viscoelastic materials described by four models of viscoelasticity for various sandwich plates in various thermal conditions will be included.

REGULAR LECTURES

Abstracts

MATHEMATICAL MODEL OF AN AXIAL MAGNETIC SPRING AND THE INFLUENCE OF ITS PARAMETERS ON STIFFNESS CHARACTERISTICS

Maksymilian BEDNAREK [1*], **Bipin BALARAM**[1], **Donat LEWANDOWSKI**[1], **Jan AWREJCEWICZ**[1]

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ABSTRACT

This study presents a mathematical model (shown in Fig. 1) and analysis of its parameters of an axial magnetic spring (depicted in Fig. 2). Unlike traditional mechanical springs, the proposed design utilizes the repulsive forces between permanent magnets to provide a highly nonlinear, hardening stiffness. The mathematical modeling is grounded in an empirical formulation of the magnetic interaction force F_m shown in eq. (1):

$$F_m = A \cdot ((-x + d + 2b)^{-2} - (x + d + 2b)^{-2}). \tag{1}$$

The eq. (1) is derived from the formula representing the axial magnetic interactions between magnets shown in [1]. The resultant restoring force of the magnetic spring, is described as a function of displacement, incorporating parameters such as the maximum interaction force between a magnet pair, the magnet half-length b, and the equilibrium distance d. Furthermore, polynomial approximations utilizing Taylor series expansion up to the 7th degree were derived to analyze the influence of specific mathematical terms. The analysis reveals that while the stiffness behaves almost linearly near the equilibrium position, higher-order terms dominate at larger displacements, confirming strong hardening characteristics. Free vibration responses of the system demonstrated good agreement between the numerical simulations and experimental data. The study presented the analysis of influence of the parameters b and d on the character of the force.

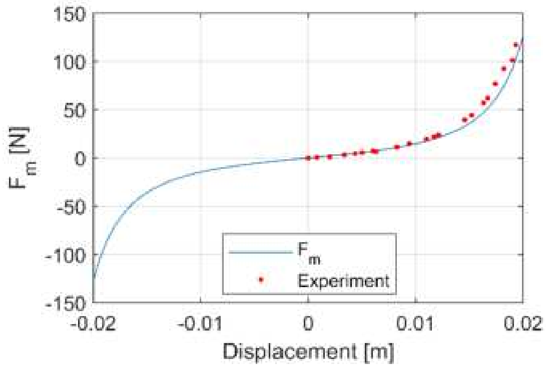

Fig. 1. Chart representing force (F_m) vs displacement with experimental data.

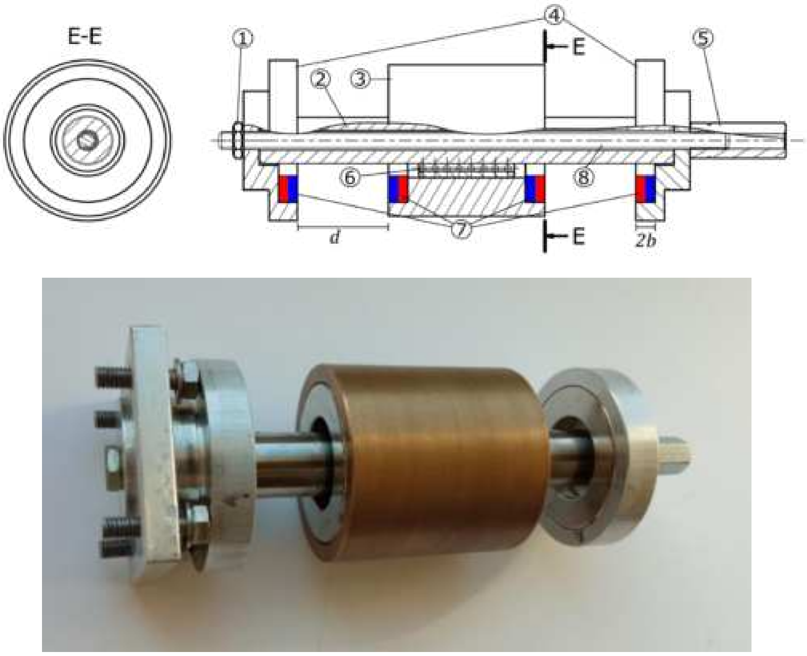

Fig. 2. Scheme and picture of the magnetic spring.

Acknowledgments

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VIBRATION ANALYSIS OF A CONTAINER FOR TRANSPORTING SENSITIVE CARGO

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ABSTRACT

The study analyzed a container design consisting of an outer and an inner frame connected by wire-rope shock isolators. The considered system is a continuation of previous works concerning the modeling of impact phenomena occurring during the transport of specialized loads sensitive to overloads [1, 2]. The main objective of the research was to determine the natural frequencies, damped vibration frequencies, and frequency ranges in which the system's dynamic response may increase.

To simplify the system's dynamic description, a two-degree-of-freedom model (Fig. 1a) was adopted. The outer frame and the inner frame with the payload were replaced by lumped masses. The wire-rope isolators were described using equivalent stiffness and damping. Additionally, the contact between the outer frame and the ground was modeled as a spring and a damper, with parameters dependent on the type of ground.

The discrete model was verified through a numerical analysis in Ansys Mechanical. The use of the Ansys environment enabled reproducing the dynamic phenomena during contact between the container and the ground, including its nonlinear and spatial nature, and obtaining additional local vibration modes of the structure.

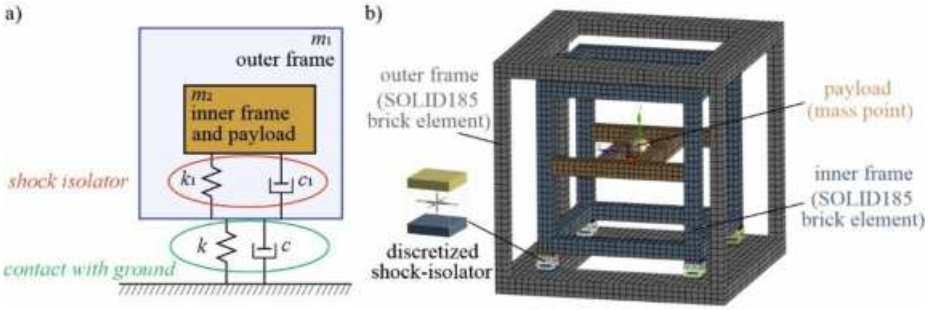

Fig. 1. A container for transporting sensitive cargo: a) a two-degree-of-freedom model, b) FEM model.

It was shown that the inner mass and the isolator parameters primarily determine the system's first natural frequency, whereas the outer mass and ground contact conditions significantly influence the higher frequencies.

The comparison of results from both models confirmed the usefulness of the two-degree-of-freedom model for the preliminary assessment of the transport container's dynamic properties. Although the simplified model assumes motion in only one direction, it correctly captures the basic relationships among the mass of the internal system, the isolator stiffness, and the ground-contact parameters. The agreement between the trends in the results confirms that the mathematical model can be used as a tool for rapid parametric analysis and the preliminary selection of system parameters.

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INFLUENCE OF LOCAL MODIFICATIONS OF METAL FACE SHEETS ON THE MECHANICAL AND DYNAMIC PROPERTIES OF MULTIFUNCTIONAL SANDWICH PANELS

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ABSTRACT

Current requirements for building envelope components lead to the search for multifunctional solutions in which mechanical performance is combined with additional functional properties, including improved acoustic performance. Previous studies in this field have shown that appropriately designed perforations can significantly improve sound absorption and the overall acoustic performance of lightweight sandwich structures [1]. However, the introduction of local discontinuities in the face sheets changes the stiffness distribution of the panel, which may affect load transfer mechanisms, stress concentration, and the dynamic characteristics of the entire structure [2].
This study investigates the influence of local modifications of metal face sheets on the mechanical and dynamic properties of multifunctional sandwich panels. Particular attention is devoted to assessing the applicability of equivalent models describing perforated regions using effective mechanical properties. Such an approach enables the analysis of entire structural components without explicitly reproducing the actual geometry of all perforation openings, while simultaneously reducing the computational cost of numerical simulations.
The investigations will be carried out using a numerical model developed in the Abaqus environment. Perforated regions will be described by equivalent mechanical parameters corresponding to the local reduction in face-sheet stiffness. The analysis will include an assessment of the static and dynamic response of sandwich panels with locally modified face sheets. The numerical investigations will involve the analysis of displacements, stress distributions, and modal characteristics to evaluate changes in natural frequencies and corresponding mode shapes caused by perforations. Modal analysis will provide insight into how local stiffness changes in the face sheets resulting from perforations affect the dynamic behavior of the entire structure.
The planned investigations will include a comparison between reference panels without perforations and panels with various configurations of locally modified face sheets. The effects of the core type as well as the geometric and mechanical parameters of the perforated regions on the structural response will be analyzed.
The obtained results may provide a basis for the design of multifunctional sandwich panels with controlled mechanical, dynamic, and functional properties.

Acknowledgments
This work was funded by the Poznań University of Technology - 0411/SBAD/0014.

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STEADY-STATE NONLINEAR VIBRATIONS OF A CANTILEVER BEAM WITH AN ECCENTRIC MASS AT THE END AND FRACTIONAL DAMPING

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ABSTRACT

The paper presents nonlinear steady-state vibrations of a cantilever beam, with a rigid mass element attached at its free end, whose center of mass does not coincide with the point of its attachment (Fig. 1).

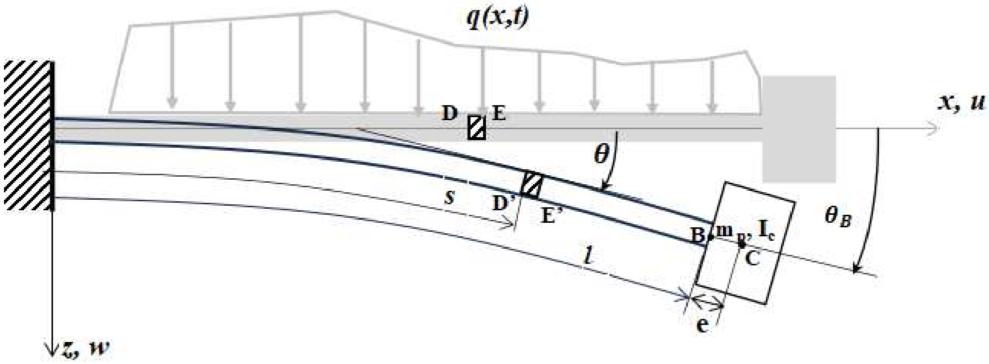

Fig. 1. Schematic of the analyzed beam.

The Euler-Bernoulli beam theory and the fractional viscoelastic Kelvin-Voigt model of beam material are assumed, where the stress-strain relationship is described as below

$$\sigma(t) = E\big(\varepsilon(t) + \mu_\gamma\, D^{(\gamma)}(\varepsilon(t))\big) \tag{1}$$

Where $\sigma(t)$ and $\varepsilon(t)$ are the stress and strain functions of time, μ_γ is a time constant [1], E is the relaxed modulus, t is time and $D^{(\gamma)}(\cdot)$ is the Caputo fractional derivative of the order $0 < \gamma \leq 0$, formulated as

$$D^{(\gamma)}\big(f(t)\big) \equiv \frac{d^\gamma}{dt^\gamma}\big(f(t)\big) \equiv \frac{1}{\Gamma(\mathfrak{m}-\gamma)} \int_0^t \frac{D^{(n)}\big(f(\tau)\big)}{(t-\tau)^{\gamma+1-\mathfrak{m}}}\, d\tau \tag{2}$$

The beam nonlinear equation of motion is derived using Hamilton's principle. The characteristic equation, modal frequencies, eigenfunction and orthogonality condition are obtained for the linear beam vibrations. The multiple scales method is used to solve the equation of motion of the beam in the vicinity of the first resonance. The first eigenfunction of linear vibrations is used as an approximate solution for the nonlinear vibrations. The impact of order of the fractional derivative on the beam vibrations is studied. The performed analysis shows that the order of the fractional derivative affects the nonlinear oscillation. An increase in the order of the fractional derivative reduces the amplitude of the beam vibrations.

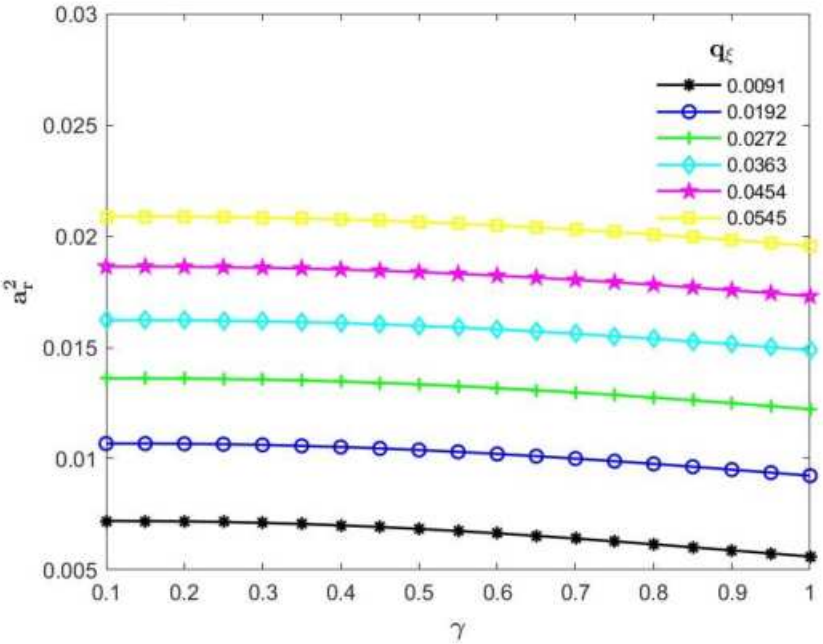

Fig. 2. The effect of the order of the fractional derivative γon the amplitude a_r^2 for $\mu_\gamma = 0.016$ and various values of the excitation force q_ξ

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THE INFLUENCE OF THE NUMBER OF STRUCTURE ELEMENTS IN A QUASI ONE–DIMENSIONAL PHONONIC CRYSTAL ON THE TRANSMISSION

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ABSTRACT

Phononic crystals are artificially designed composite materials characterized by a periodic structure. Their most important feature is the ability to create so–called acoustic bandgaps, i.e., frequency ranges where sound wave propagation within the structure is completely blocked. Quasi–one–dimensional systems provide the foundation for research into this phenomenon, combining the simplicity of mathematical description with enormous application potential. Understanding the mechanisms of bandgap formation in such structures allows for effective noise suppression and precise control of acoustic energy flow in modern engineering systems.

A key factor determining the performance of a phononic crystal is the number of repeating components that constitute its periodic structure. While an infinite ideal crystal guarantees full attenuation in the bandgap, real finite–length systems exhibit a strong dependence of the depth and sharpness of the bandgap boundaries on the number of unit cells. Investigating the minimum and optimal structure size is crucial for device miniaturization and material cost reduction. This work focuses on analyzing the effect of the number of components in a quasi–one–dimensional phononic crystal on the final acoustic wave transmission characteristics.

The Transfer Matrix Method (TMM) was used in this work. This algorithm is a well–established and highly efficient numerical tool for analyzing wave propagation in quasi–one–dimensional systems. Using TMM, numerical simulations of the transmission coefficient were performed for a sequence of crystals composed of various numbers of unit cells. These calculations allowed for precise tracking of the bandgap evolution – from its first symptoms with only a few elements to the full formation of a highly attenuated bandgap. The experimental part of the work involved the physical fabrication of the designed structures using 3D printing technology. Polylactide (PLA) was used as the building material, which, due to its availability, stiffness, and repeatable mechanical parameters, is ideal for modeling acoustic barriers. The additive method allowed for the rapid and precise production of a series of samples differing only in the number of repeatable structural segments. This approach ensured high geometric repeatability, eliminating the influence of other macroscopic factors on the final transmission measurement result.

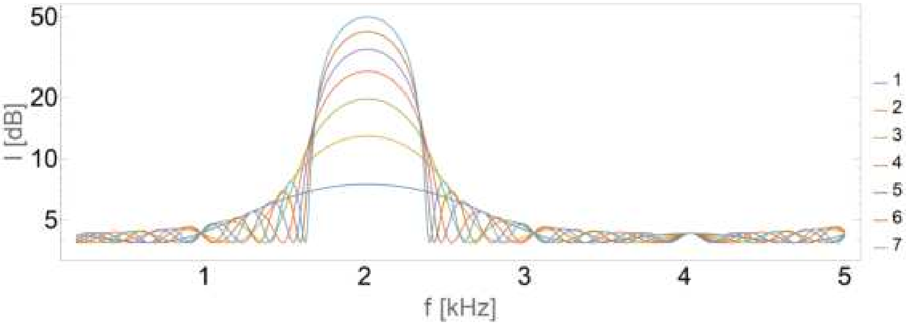

Fig. 1. Transmission loss amplitude as a function of frequency (numerical analysis).

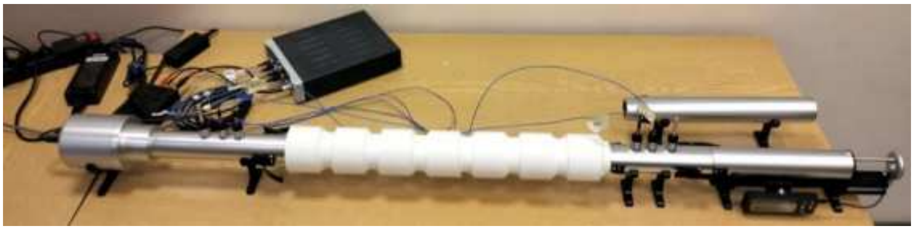

Fig. 2. Measuring station with a tube.

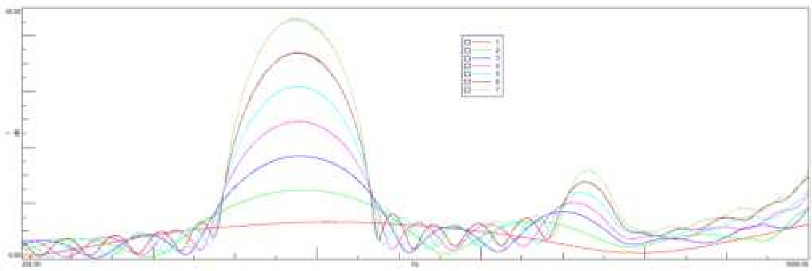

Fig. 3. Transmission loss amplitude as a function of frequency (experiment).

The numerical assumptions (Fig. 1) were verified on a laboratory bench equipped with an impedance tube (Fig. 2), model TUBE-X 34.9 mm diameter, SMSL SA36A PRO power amplifier, Simcenter SCADAS SCM2E01 4-channel analyzer. Acoustic transmission measurements (Fig. 3) were performed using the transfer function method with a set of measurement microphones (model 378A14 from PCB Piezotronics Inc.), which allowed for the determination of actual attenuation characteristics as a function of frequency. Experimental results confirmed high agreement with TMM models, demonstrating a logarithmic increase in attenuation within the band gap with the addition of subsequent structural elements.

Acknowledgments
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DYNAMICS OF NONLINEAR OSCILLATORS COUPLED BY MAGNETIC FIELDS

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ABSTRACT

Modelling complex dynamical systems is a critical aspect in prototyping new engineering solutions, enabling the verification of device functionality and cost reduction. Recently, coupled systems have gained popularity, particularly in applications related to vibration suppression and mechanical energy harvesting. This research investigates a unique configuration of two nonlinear magneto-electro-mechanical oscillators coupled at a distance. Unlike traditional mechanical coupling methods, the interaction between these systems is achieved through magnetic fields generated by electric coils connected in series, which share a common electric current [1-2].

The investigated 2-degree-of-freedom (2-DoF) system consists of two identical 1-DoF oscillators. To ensure precise, friction-free linear motion of the oscillating masses, the experimental setup utilizes aerostatic bearings. Each individual oscillator features a "magnetic spring" created by two opposing neodymium permanent magnets oriented to provide a repulsive interaction. A key methodological contribution of this work is the successful adaptation of an existing analytical model, originally describing the interaction between a permanent magnet and a single-layer coil, to accurately describe the multi-layer coils used in the constructed system.

The physical model of the single considered oscillator consisting of a body with mass m vibrating in a gravitational field with a coefficient of gravity g. Position x of the oscillator is measured by the distance between the frontal surfaces of two neodymium permanent magnets. The mass m is "suspended" by the magnetic force $F_m(x)$ appearing between them. This pair of magnets serves as the magnetic spring in the system. The oscillating mass m includes an additional neodymium magnet, which can move in the hollow of the electric coil and produces an additional force $F_c(x,\dot{x})$ acting on this mass. The movement of the body with mass m along the x direction takes place in aerostatic bearing. Therefore, the force of dry friction can be neglected and only the viscous friction force proportional to the velocity $\dot{x}$, with the viscous friction coefficient c, can be considered. The studied system also takes into account harmonic excitation in the form of a mass m_0 rotating with an angular velocity ω on a radius (unbalance) e.

The dynamics of the coupled 2-DoF system, where the first oscillator is harmonically forced by a rotating unbalance mass and the second remains unforced, are governed by the Eq. 1:

$$\begin{cases} (m+m_0)\ddot{x}_1 + c\dot{x}_1 + (m+m_0)g + F_{c1}(x_1,\dot{x}_1,x_2,\dot{x}_2) = F_{m1}(x_1) + m_0 e\omega^2 \sin\omega t, \\ (m+m_0)\ddot{x}_2 + c\dot{x}_2 + (m+m_0)g + F_{c2}(x_1,\dot{x}_1,x_2,\dot{x}_2) = F_{m2}(x_2). \end{cases} \tag{1}$$

The coupling mechanism is embedded in the magnetic interaction forces F_{c1} and F_{c2}, which dynamically depend on the states (position and velocity) of both oscillators simultaneously.

Detailed numerical analysis was conducted to study the steady-state dynamics, utilizing time histories, phase portraits, and frequency characteristics. These numerical results were experimentally verified using a custom-built experimental stand equipped with a data acquisition system. Both the simulations and the experimental data revealed a distinct hysteresis phenomenon. The system exhibits different amplitude-frequency response curves depending on whether the harmonic excitation frequency is increased or decreased, as presented in Fig. 1.

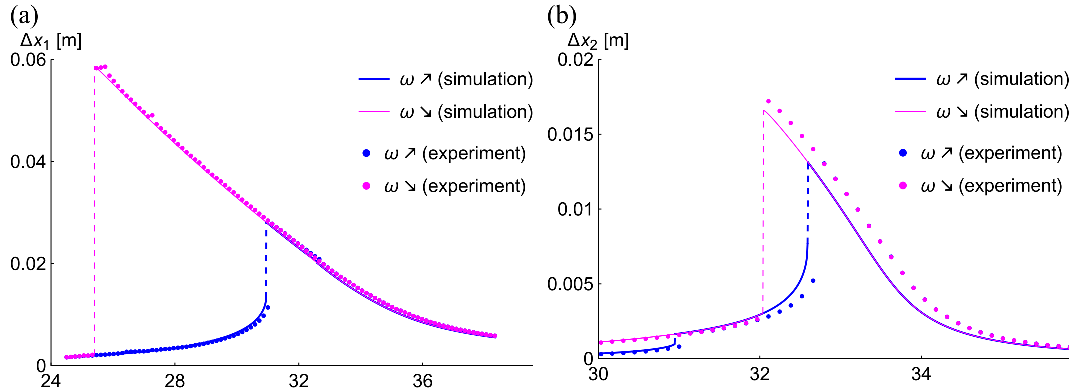

Fig. 1. Frequency characteristics of the vibration amplitudes Δx_1 (a) and Δx_2 (b) in the steady state: simulation vs. experiment.

The model of interaction between a permanent magnet and a single-layer electric coil existing in the literature is successfully adapted and applied to the interaction between magnet and multi-layer coil. The study demonstrates a strong agreement between numerical predictions and experimental measurements. This accuracy is particularly notable in predicting the values and shapes of the generated electromotive force and electrical power, outperforming simpler empirical models. In the context of energy harvesting, the observed hysteresis effect offers a strategic advantage. The optimal energy harvesting scenario involves gradually decreasing the excitation frequency, which leads to a continuous increase in the generated electrical power. However, this reduction must be carefully managed to avoid reaching the critical frequency point where a sudden drop in vibration amplitude causes rapid energy dissipation. The proposed coupled oscillator design shows broad potential for practical applications in vibration mitigation and energy accumulation strategies.

Acknowledgments
This work has been supported by the Polish National Science Centre under the grant OPUS 18 No. 2019/35/B/ST8/00980.

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TOWARDS REALISTIC DYNAMIC MODELING OF SURGICAL ROBOTS: AN RCM MECHANISM BASED ON EXPERIMENTAL DATA FROM CARDIOVASCULAR TISSUES

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ABSTRACT

Dynamic modeling and control of surgical robots equipped with a Remote Center of Motion (RCM) mechanism are of significant importance in minimally invasive surgery. In this study, motion data obtained from *in vitro* experiments on cardiovascular tissues performed by an experienced cardiac surgeon were utilized.

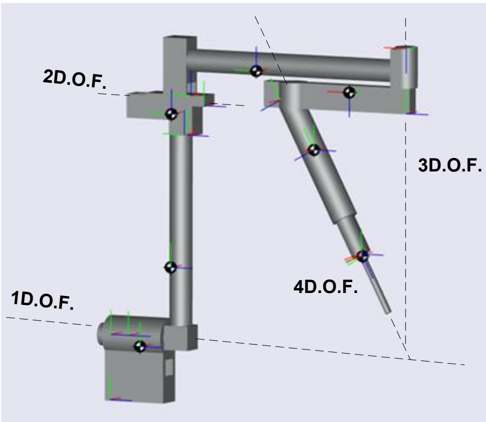

Fig. 1. Surgical robot model with the RCM mechanism made in the Simulink environment

A dynamic model of an RCM-based surgical robot was developed in the MATLAB/Simulink environment using a block-diagram approach. Robot joint control was implemented using a conventional PID controller, a fuzzy PID controller, and an adaptive PID controller.
A comparative analysis was conducted using realistic motion trajectories, focusing on trajectory-tracking accuracy and dynamic performance. The obtained results demonstrate the suitability of the proposed model for the design and simulation of surgical robotic systems intended for minimally invasive surgical applications.

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INTERVAL GENERALIZED FINITE DIFFERENCE METHOD FOR SOLVING THE POISSON EQUATION – THE FIRST APPROACH

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ABSTRACT

We consider the interval formulation of the generalized finite difference method (GFDM) for solving the Poisson equation. The interval approach based on the interval arithmetic [4] is of special interest because we can include in the final solution all errors that arise during computations, i.e., the errors caused by the floating-point arithmetic (representation errors and rounding errors) and the errors that are due to inexact initial data. In the second case the values can be enclosed in an appropriate intervals which endpoints depend on the measurement uncertainties. Finally, if we consider classical or generalized finite difference methods a truncation error occurs and is further neglected in their final formulations. In the interval approach we try to include this error of the method in the final interval solution. The paper concerns the generalized finite difference methods [2-3, 5-8] and their interval counterparts due to the fact that they can be easily applied to irregular grids (clouds) of points. In many engineering applications the domain has complicated boundary and for this reason such an approach is important to develop. We propose the interval generalized finite differences [1] and then the interval generalized finite difference method for solving a boundary value problem with the Poisson equation.

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NONLINEAR DYNAMICS OF A PARAMETRICALLY EXCITED 2DOF OSCILLATOR WITH MAGNETIC RESTORING FORCE AND DRY FRICTION

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ABSTRACT

This work presents an analytical and numerical investigation of a nonlinear two-degree-of-freedom mechanical oscillator subjected to parametric excitation, magnetic stiffness nonlinearity, and dry friction. The proposed system consists of two coupled oscillating masses connected through a rotating rectangular beam, whose directional stiffness produces time-periodic parametric excitation. Strong nonlinear restoring forces are introduced through repulsive magnetic springs, while dry friction is considered to represent motion resistance in the guiding mechanism.

The complexification-averaging method is employed to derive approximate analytical modulation equations and amplitude–frequency responses of the system. The obtained analytical solutions are validated through direct numerical simulations using the Runge–Kutta method. To examine the system dynamics in detail, bifurcation diagrams, phase portraits, Poincaré maps, and Lyapunov exponents are computed for both equal- and unequal-mass configurations.

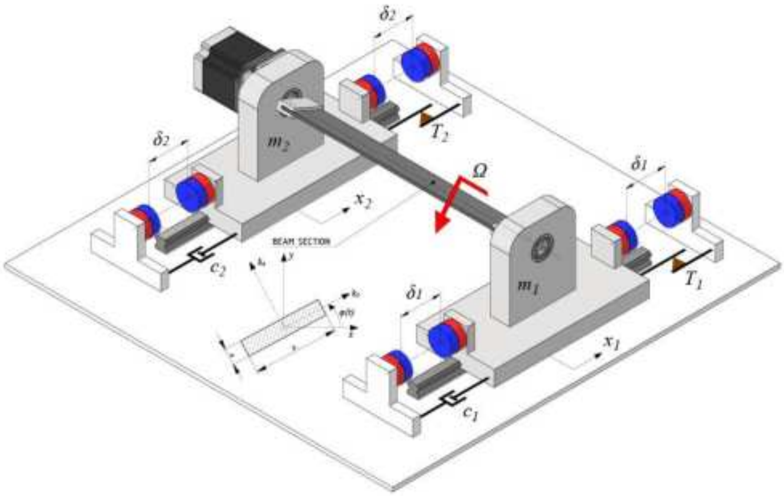

Fig. 1. Physical concept of 2DOF mechanical parametric oscillator.

The results show that the 2DOF coupled parametric oscillator exhibits a rich variety of nonlinear responses, including periodic, quasi-periodic, and chaotic motion. For unequal masses, the response is mainly periodic and closely resembles the behaviour of the corresponding single-degree-of-freedom system [1, 2]. In contrast, the equal-mass configuration produces more complex bifurcation patterns, multistability, and transitions to quasi-periodic and chaotic regimes. The analysis also demonstrates that magnetic nonlinearity, dry friction, and mass distribution significantly influence resonance behaviour, stability, and vibration amplitude [3]. These findings provide useful insight into the dynamics of coupled nonlinear mechanical systems and support the design of advanced vibration mitigation devices, nonlinear energy harvesters, and mechanical systems with tunable stiffness characteristics.

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STABILITY AND FREE VIBRATIONS OF GEOMETRICALLY NONLINEAR COLUMN UNDER SPECIFIC LOAD IN ASPECT OF INTERNAL CRACK OCCURENCE

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ABSTRACT

This paper presents a comprehensive analysis of the stability and free vibrations of a geometrically nonlinear column subjected to conservative axial loading, with particular emphasis on the influence of an internal crack. The column under consideration is composed of three slender rods with defined asymmetry in flexural stiffness and subjected to a selected specific load (Tomski's load). Cracks are simulated using pins and rotational springs, thus ensuring the equality of transverse displacements, longitudinal forces, bending moments and deflection angles at their locations [1]. The aim of this work is to quantify how the presence, location, and depth of a crack affect the bifurcation force (loss of rectilinear form of static equilibrium) and the natural frequencies of the column.

In this paper, a mathematical model based on the energy method was developed. The problem was formulated based on Hamilton's principle. The differential equations of motion were derived taking into account geometric nonlinearity and solved using the small amplitude parameter method [2-3]. The problem was solved numerically to determine the values of bifurcation forces and changes in natural frequencies as a function of external load.

Based on numerical calculations, the relationship between the depth and location of the crack along the length of the column and the corresponding changes in its behaviour, including the local loss of rectilinear static equilibrium, was investigated. Depending on the values of the assumed geometric and physical parameters of the column, the variability of the bifurcation load and the first natural frequencies were determined. The test results provide valuable information for monitoring the technical condition of the structure, detecting damage, and assessing the safety of slender structural elements.

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MODAL PARAMETERS OF A WIND TURBINE BLADE UNDER VARIABLE TEMPERATURE CONDITIONS: IDENTIFICATION FROM FULL-SCALE LABORATORY TESTS

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ABSTRACT

Temperature-related stiffness changes in composite wind-turbine blades may affect modal parameters and, if neglected, complicate damage assessment. This study investigates the influence of temperature on the dynamic response of a full-scale blade and evaluates an automated operational modal analysis (Auto-OMA) procedure using one analysis configuration for all test conditions. Earlier static tests of the same 12.6 m blade documented a repeatable increase in stiffness during cooling [1].

Fig. 1. Full-scale laboratory setup: the wind-turbine blade was mounted in the steel-box root support inside the climatic chamber.

The blade was tested in a climatic chamber at +20, 0 and −20 °C under broadband random excitation. Four measurement setups with common reference sensors were used to cover the blade length with 0.5 m spacing. Each temperature was processed using the same configuration in three sampling-rate branches (50, 100 and 200 Hz), giving 36 setup–temperature–sampling-rate cases.
Auto-OMA was based on singular-value decomposition of the output spectral-density matrix and uncertainty-aware covariance-driven stochastic subspace identification (SSI-COV) [2,3]. Stable poles were screened using a Gaussian-mixture model and grouped into modal cores with HDBSCAN. The final ranking combined stability, spectral, modal-shape, damping and uncertainty information in a modal-evidence factor graph. Local results from individual setups were merged using PoSER [4]. Robustness was assessed through a post-SSI sensitivity analysis, and modes were matched between temperatures using frequency proximity and the modal assurance criterion (MAC). The procedure is consistent with

the principles of fully automated OMA [5].

Across all three temperatures, the workflow returned 37 modal estimates: 14 at +20 °C, 12 at 0 °C and 11 at −20 °C. Eleven modal families were identified at every temperature. Their natural frequencies at +20 °C ranged from 2.15 to 64.88 Hz. On cooling from +20 to −20 °C, all complete families exhibited an increase in frequency of 0.59–2.99%, while the corresponding cross-temperature MAC values ranged from 0.952 to 1.000. Figure 2 presents the reference SV1 spectrum and the frequency trends. Families observed at only one or two temperatures are indicated by dashed lines in Fig. 2(a) and are omitted from Fig. 2(b). The measured temperature dependence should be considered when modal features are used for blade condition assessment.

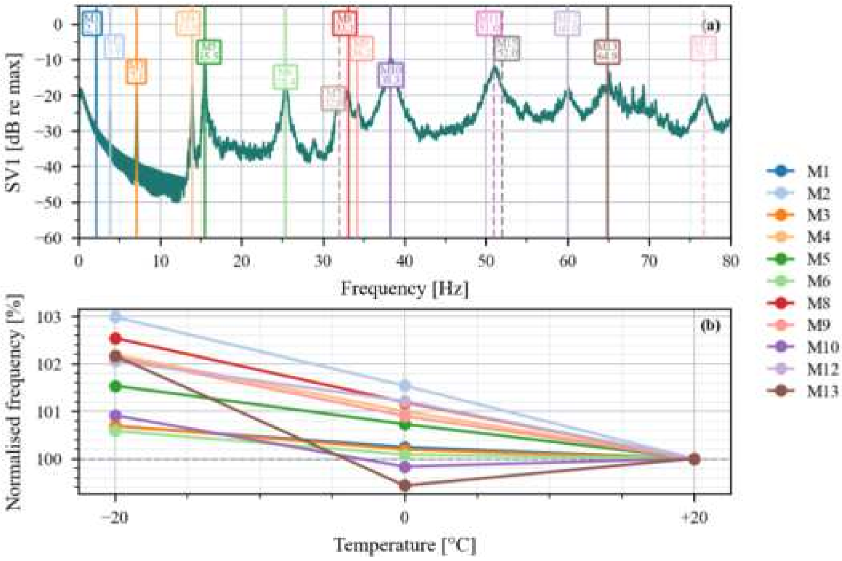

Fig. 2. (a) Reference SV1 spectrum at +20 °C. Solid and dashed vertical lines denote modal families identified at all three temperatures and at one or two temperatures, respectively. (b) Relative natural-frequency changes normalised to +20 °C for the complete modal families.

Acknowledgments

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STIFFNESS OF A WIND TURBINE BLADE UNDER VARIABLE TEMPERATURE CONDITIONS: EXPERIMENTAL INVESTIGATION AT MATERIAL AND STRUCTURAL SCALES

Przemysław KALITOWSKI[1], Beata ZIMA[2], Arkadiusz JENTA[3], Marcin KRAJEWSKI[4], Agnieszka TOMASZEWSKA[5], Michał WÓJCIK[6]

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ABSTRACT

Wind turbines operating in cold climates are exposed to significant seasonal temperature variations that influence the mechanical response of composite blades. Since many structural health monitoring (SHM) and digital twin approaches rely on changes in stiffness-related parameters, understanding the temperature dependence of blade stiffness is essential for reliable structural assessment.

This study presents an experimental investigation of the influence of temperature on the stiffness of a 12.6 m composite wind turbine blade at two different structural scales. Material-level characterization was performed using tensile tests on composite specimens extracted directly from the blade and tested at temperatures ranging from +20°C to -20°C. In parallel, full-scale static tests were conducted inside a dedicated large-size climate chamber under identical thermal conditions. The blade was subjected to quasi-static flap-wise loading, while its response was monitored using strain gauges, displacement sensors, and temperature measurements.

The experimental results demonstrate a clear increase in stiffness with decreasing temperature at both scales. However, the magnitude and clarity of the observed trends differ considerably. Material tests revealed substantial scatter caused by the inherent variability of specimens extracted from a real blade, although an overall increase in Young's modulus with decreasing temperature was identified. In contrast, the full-scale blade exhibited a highly repeatable and nearly linear stiffness-temperature relationship, with an average increase in global stiffness of approximately 2.7% between +20°C and -20°C. The comparison highlights that coupon-level material properties cannot be directly translated into the structural response of the complete blade, as the latter reflects load redistribution and structural averaging effects. The obtained results indicate that temperature compensation should be

incorporated into stiffness-based diagnostics, structural health monitoring systems, and digital twin models dedicated to wind turbine blades operating in cold climates.

Acknowledgments
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FROM EQUATIONS TO INTUITION: ACTIVE LEARNING STRATEGIES IN TEACHING MECHANICS AND STRENGTH OF MATERIALS FOR CIVIL ENGINEERING STUDENTS

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ABSTRACT

Mechanics and Strength of materials are foundational yet notoriously challenging subjects for civil engineering undergraduates. Students frequently struggle with the gap between abstract theoretical formulas and real engineering behavior, and with developing genuine physical intuition alongside procedural problem-solving skills. Research consistently shows that active learning strategies significantly outperform traditional lecturing in undergraduate STEM (Science, Technology, Engineering and Science) courses, both in examination performance and in student retention [1, 2].

This paper presents a multi-method didactic approach developed and applied in a civil engineering program, combining seven complementary active learning strategies. The course is built around a traditional lecture as the informational backbone, supplemented by exercise sessions focused on structured problem-solving. Beyond these standard elements, the approach incorporates: (1) a project method, in which students independently design and analyze structural elements; (2) laboratory experiments providing direct observation of material behavior under load; (3) individual student presentations of solved problems to peers, fostering communication skills and deeper understanding; (4) escape room sessions, sets of review problems presented as a time-limited team challenge, an approach shown to enhance engagement and problem-solving skills in STEM higher education [3]; and (5) analysis of real structural failure case studies, which effectively blend theoretical principles with real-world engineering decisions and complement lecture-based instruction [4].

A particularly valued component consists of conceptual reasoning problems, namely tasks requiring physical intuition and qualitative thinking rather than formula application. These problems reveal and challenge students' mental models of mechanical phenomena, directly addressing the equation-to-intuition gap named in the title.

The paper discusses the rationale for each method, their integration within the course structure, and observed effects on student engagement and understanding. The aim is to share transferable practices with educators in mechanics and related engineering disciplines.

Acknowledgments

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ACOUSTIC EFFICIENCY OF ENCLOSURES WITH TRANSPARENT WALLS

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ABSTRACT

In addition to reducing noise levels, the enclosures with transparent walls are designed to ensure monitoring of the machine or device. The aim of the research, conducted on a prototype sound-insulating enclosure with glass panels mounted in a frame made of profiles manufactured using 3D printing technology, was to assess the acoustic effectiveness of the proposed solution. Preliminary research utilized a survey method to determine the sound power levels of an unenclosed and enclosed loudspeaker imitating a noisy machine or device. Based on the difference in these levels, the enclosure's insertion loss was determined. To improve the acoustic effectiveness of the prototype enclosure, two further material and design solutions were proposed. The first variant involved utilizing the effect of double glass walls, while the second involved the use of sound-absorbing material between these baffles. Calculations of the transmission loss of individual baffles were carried out using INSUL sound insulation prediction software. Acoustic tests of the proposed new variants of material and construction solutions for the enclosure showed an improvement in acoustic efficiency, the greatest in the frequency range of 250-500 Hz and above 2500 Hz.

Acknowledgments
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POST-CRITICAL PHENOMENA IN MECHANICAL SYSTEMS WITH FOLLOWER LOADS

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ABSTRACT

Mechanical systems subjected to non-conservative forces, particularly follower loads, constitute a classical and continuously relevant problem in machine dynamics and structural mechanics. The loss of stability in such systems typically leads to divergence (static instability) or flutter (dynamic instability) phenomena. While the linear stability analysis of these phenomena is well documented in the literature, the post-critical behavior of the system- especially in the presence of strong nonlinearities- remains a research challenge.

This study focuses on the analysis of complex post-critical phenomena in systems with follower loads, incorporating the nonlinear effect of dry friction. Dry friction plays an ambiguous role in non-conservative systems; on the one hand, it acts as an energy dissipation mechanism, but on the other hand, through a destabilizing effect (Ziegler's paradox), it can significantly lower the critical load value and lead to counterintuitive changes in the system's dynamics. The objective of this research is to identify the mechanisms governing the system's response following the loss of stability of the trivial equilibrium position.

Within the scope of this work, selected phenomenological models will be subjected to analytical and numerical investigations to evaluate the impact of friction on post-critical dynamics. The methodology encompasses the determination of bifurcation paths, the analysis of phase space topology, and the direct integration of the equations of motion.

Particular attention will be paid to assessing the influence of friction parameters on the nature of the occurring bifurcations (including transitions between supercritical and subcritical bifurcations) and the formation of stable and unstable limit cycles. The study will also analyze the conditions for the emergence of complex phenomena within the flutter vibration regime. Additionally, the effect of energy dissipation due to friction on the formation of basins of attraction for coexisting solutions will be investigated, which determines the system's sensitivity to initial conditions.

The conducted analyses are expected to shed new light on the coupling between instability induced by a follower force and contact nonlinearity. These considerations hold potential cognitive and practical significance for predicting the behavior of advanced mechanical systems susceptible to flutter, allowing for a better understanding of their safety margins from a nonlinear perspective.

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DEVELOPMENT AND FEM VALIDATION OF A HENCKY-TYPE NONLINEAR BEAM MODEL

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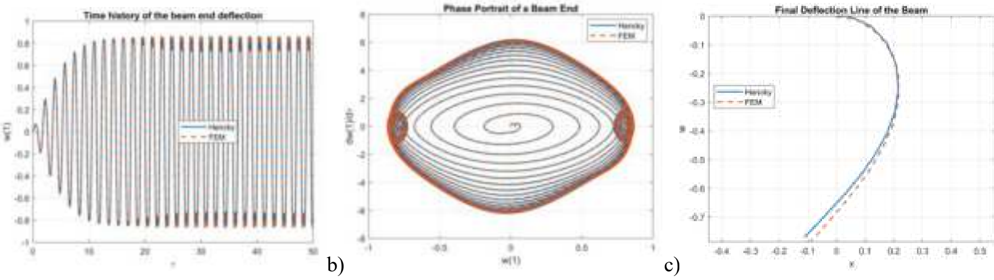
ABSTRACT

A Hencky-type discrete model is developed and validated for dynamic analysis of geometrically nonlinear slender beams [1]. The beam is represented by rigid segments connected through rotational springs and dampers, which allows large rotations and large transverse deflections to be described with a relatively small number of generalized coordinates. The governing equations are written in an explicit matrix form for an arbitrary number of elements, which makes the formulation convenient for numerical implementation and parameter studies [1]. This approach is closely related to finite-difference descriptions of discrete elastica based on the Hencky system [2]. Finite-element elastica formulations based on distributed microstructure have also been studied in the literature [3].

A key feature of the model is the unified treatment of right-end boundary conditions. Horizontal, vertical and rotational boundary springs, together with an additional axial stiffness, allow one to represent classical, non-classical and imperfect supports. In particular, the same formulation can describe fully shortened, fully stretched and intermediate partly-shortened or partly-stretched configurations [1]. This is important in nonlinear beam dynamics, since boundary flexibility may significantly modify the frequency response and may lead to transitions between softening- and hardening-type behaviour.

The validated mechanical core may be coupled with additional functional elements, such as passive viscous dampers, shear-thickening-fluid dampers, piezoelectric actuators or vibration energy-harvesting devices [4]. Therefore, the model is intended not only for isolated beam simulations, but also as a compact basis for smart beam systems in which geometric nonlinearity, support flexibility and local functional components interact.

In the validation example shown in Fig. 1, a cantilever beam subjected to a harmonic tip bending moment is analysed. The Hencky solution is compared with an independently implemented co-rotational finite element method (FEM) model using the same number of elements and comparable numerical accuracy. The comparison includes the dimensionless tip-deflection time history, the tip phase portrait and the instantaneous deformed shape.



a) b) c)

Fig. 1. FEM validation of the Hencky-type nonlinear beam model for a cantilever beam subjected to a harmonic tip bending moment: (a) dimensionless tip-deflection time history; (b) tip phase portrait; (c) instantaneous deflected shape at the final time.

Very good agreement is obtained in both the transient and steady-state parts of the response. The phase portraits practically overlap, and only a small local difference is visible near the beam tip in the final deformed configuration, consistently with the expected discretisation error. At the same time, in the tested implementation, the Hencky model reduced the computational time by about 10 times compared with FEM. This makes it particularly useful for long-time simulations, parameter sweeps, bifurcation analysis and optimisation of nonlinear damping, actuation or energy-harvesting beam systems.

Acknowledgments
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STRUCTURAL DYNAMIC IDENTIFICATION USING A STOCHASTIC METHOD OVER A GRANDSTAND STRUCTURE

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ABSTRACT

The Stochastic Subspace Identification (SSI) method is a well-established output-only dynamic system identification technique that has been widely applied in the field of Operational Modal Analysis (OMA) over the past decades [1]. The method exploits the stochastic nature of ambient vibration responses, which are assumed to be excited by broadband (approximately white noise) environmental loads, to identify the dynamic characteristics of a structure. By employing orthogonal projections and subspace decomposition, SSI enables the estimation of the system state-space model whose modal parameters are obtained by singular value decomposition. However, due to output-only data measurements and practically rough assumption on the input, the collected structural responses are considered noisy. The SSI technique provides a framework for quantifying the uncertainty associated with the identified modal parameters by a first order sensitivity analysis of the identified state and output matrices.

Human–Structure Interaction (HSI) remains one of the most challenging topics in modern structural engineering, particularly for assembly venues where synchronized crowd activities may induce excessive vibrations and potentially excite structural resonance. Several studies have documented this phenomenon in large stadiums, including the Leeds Stadium (UK) [2], the Giuseppe Meazza Stadium (Italy) [3], and the São Paulo Stadium (Brazil) [4]. The case study considered in this work corresponds to a segment of the steel grandstand structure of the Diego Armando Maradona Stadium in Naples, Italy, representing one twenty-eighth of the complete elliptical stadium. The investigated structure is composed of ASTM A992 Grade 50 steel members with overall dimensions of 20,773 × 9,830 mm². The primary and secondary beams consist of H-shaped sections measuring 1680 × 500 mm² and 1650 × 500 mm², respectively, as illustrated in Fig. 1. The grandstands are supported by steel columns, constructed separately from other facilities of the stadium. A detailed finite element model was developed in SAP2000 and calibrated using modal properties obtained from a previous experimental system identification campaign. The numerical model consists of beam-column frame elements representing the selected stadium segment and incorporates spring supports to simulate the elastomeric bearings connecting the grandstand to the main supporting columns. Particular attention was devoted to modelling structural

uncertainties associated with the mechanical properties of the rubber bearings and the stiffness of the structural connections.

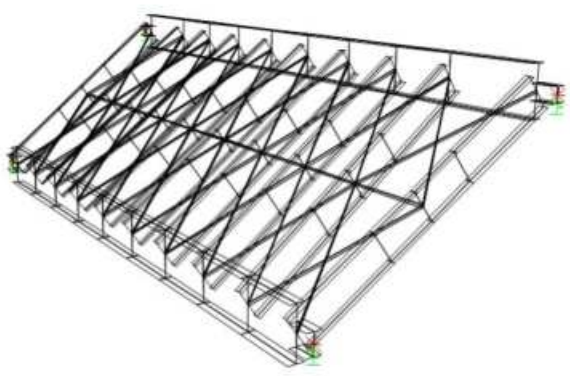

Fig. 1. SAP2000 model of grandstand steel structure.

The experimental campaign employed eight accelerometers strategically distributed over the structure, including six sensors measuring vertical accelerations and two measuring horizontal accelerations. Vibration measurements were acquired on June 26th during a live concert by the Italian contemporary artist Geolier at the Maradona Stadium. The instrumentation consisted of PCB Piezotronics Model 393B04 accelerometers, with a measurement range of ±49 m/s², a sensitivity of 102 mV/(m/s²), and a sampling frequency of 1000 Hz. The recorded acceleration signals were pre-processed in MATLAB using a zero-phase Butterworth band-pass filter with cutoff frequencies of 0.3 Hz and 30 Hz to remove low-frequency drift and high-frequency measurement noise. Subsequently, the modal identification was performed using the pyOMA framework [5]. The identified natural frequencies, mode shapes, and damping ratios exhibited good agreement with the predictions of the calibrated SAP2000 model, thereby providing experimental validation of the proposed numerical model.

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THE INFLUENCE OF THE TYPE OF DAMPING MATERIAL, OPERATING TEMPERATURE AND DAMPING ELEMENT GEOMETRY ON THE EFFECTIVENESS OF PASSIVE VIBRATION ISOLATION

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ABSTRACT

Passive vibration damping is widely used in lightweight transport structures; however, its effectiveness depends on the properties of viscoelastic materials, environmental conditions, damping configuration, and the additional structural mass introduced by damping treatments [1].

The paper presents a comprehensive numerical and experimental analysis of the effectiveness of passive vibration damping, with particular emphasis on aerospace applications. Seven beam specimens were used to investigate the influence of damping material type, damping treatment type, operating temperature and damping element geometry on the effectiveness of passive vibration isolation. The specimens were prepared in accordance with ASTM Standard E756-05 [2], and consisted of an undamped aluminium reference beam and beams treated with Free Layer Damping (FLD) and Constrained Layer Damping (CLD) using butyl rubber and bituminous materials.

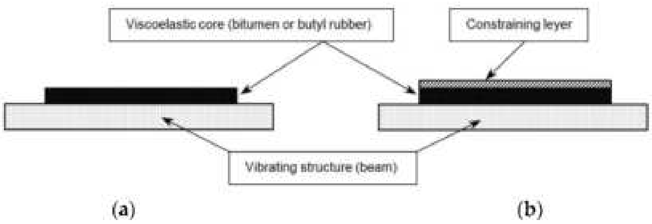
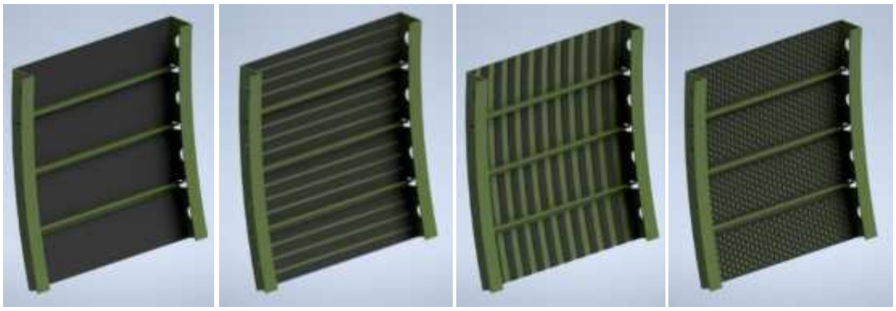


Fig. 1. Viscoelastic materials damping: (a) FLD treatment and (b) CLD treatment [1]

Using an Unholtz-Dickie UDCO TA-250 electrodynamic vibration system and a custom-designed cooling chamber [3], a series of modal tests were conducted over a temperature range from -2°C to 22°C. The cooling chamber was based on the Peltier effect. The experiments were carried out in the frequency range from 7 Hz to 4 kHz with a constant excitation level of 1 g (1 g = 9.81 m/s^2). The vibration response of the test beams was measured using piezoelectric accelerometers. Frequency Response Functions (FRFs) were measured for each beam specimen. Subsequently, the loss factor (η) corresponding to the considered vibration modes was calculated using the n-dB method [2]. Since a high value of the loss factor (η) indicates good energy dissipation capability of the structure, the damping performance of the sample could be assessed.

The experimental investigations revealed a significant influence of operating temperature on the damping effectiveness of the analysed viscoelastic materials. Depending on the material type and damping treatment type, substantial variations in energy dissipation capability were observed throughout the investigated temperature range. The results also confirmed that the effectiveness of passive damping treatments cannot be assessed independently of their operating environment, particularly in applications exposed to varying thermal conditions.

A significant contribution of the study is the experimental determination and validation of frequency-dependent viscoelastic material properties required for finite element simulations of damped structures. The variation of the loss factor (η) and storage modulus (E′) with frequency was evaluated using data obtained from experimental modal analysis. FRFs were acquired for beam specimens with progressively reduced lengths, allowing the resonance characteristics to be determined over a wide frequency range. The loss factor (η) and storage modulus (E′) were subsequently implemented in finite element models developed in ANSYS Mechanical. The accuracy of the adopted material representation was verified through modal and harmonic response analyses by comparing numerical predictions with experimental results, establishing a validated experimental-numerical framework for the assessment of passive damping treatments.



(a) (b) (c) (d)

Fig. 2. Types of surface damping treatment of the fuselage skin: (a) full coverage, (b) horizontal strips, (c) vertical strips, (d) honeycomb pattern.

The validated model of butyl rubber and bituminous materials was then applied to a representative section of a Bombardier Dash-8 Q400 aircraft fuselage to investigate the influence of damping layer geometry on vibration isolation. Four types of surface coverage were analysed: full coverage, horizontal strips, vertical strips, and a honeycomb structure. A key outcome of the study is the development of two original mass-efficiency indicators that simultaneously quantify vibration damping and the associated structural mass increase. Unlike conventional approaches focused solely on vibration reduction, the proposed criteria enable a comparative evaluation of alternative damping layouts under weight-sensitive design constraints characteristic of aerospace structures.

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INFLUENCE OF CROSS-SECTIONAL GEOMETRY ON THE LOW-FREQUENCY VIBRATION ISOLATION FUNCTION OF 3D-PRINTED PET-G ARCHITECTED STRUCTURES

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ABSTRACT

Architected materials have recently gained attention due to their tuneable dynamic and vibration-isolation properties [1-2]. This study experimentally investigates the influence of cross-sectional geometry on the low-frequency vibration-isolation properties of 3D-printed PET-G architected structures. Specimens (Fig. 1) with comparable wall thicknesses and different internal geometries, including omega-shaped, lightning-shaped, right-angle, and chevron patterns, were manufactured using fused deposition modelling. Each specimen was mounted between an electrodynamic vibration exciter and a payload with a total mass of 0.7285 kg. Harmonic excitation was applied over a frequency range of 11-200 Hz. Acceleration signals were recorded simultaneously on the payload and the exciter base. The vibration isolation function was determined as the ratio of the acceleration amplitude measured on the payload to that measured on the exciter base.

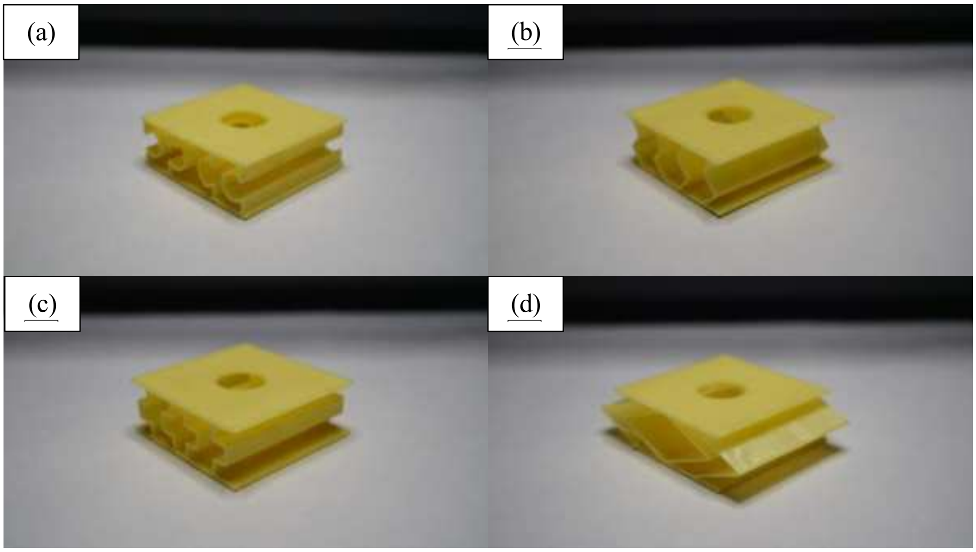

Fig. 1. Specimens with comparable wall thicknesses and different internal geometries including omega-shaped (a), lightning-shaped (b), right-angle (c), and chevron patterns (d).

The measured resonance frequencies ranged from approximately 11 Hz to 55 Hz, demonstrating a strong influence of structural geometry on the effective dynamic stiffness. A particularly clear relationship was observed for the chevron-shaped specimens. Increasing the angle from 45° and 60° to 90° and 120° shifted the resonance frequency from approximately 11–12 Hz to 26 Hz and 55 Hz, respectively. At 100 Hz, all investigated structures exhibited vibration isolation function values below unity, ranging from approximately 0.03 to 0.65, which indicates that vibration isolation was achieved at this frequency – Fig. 2.

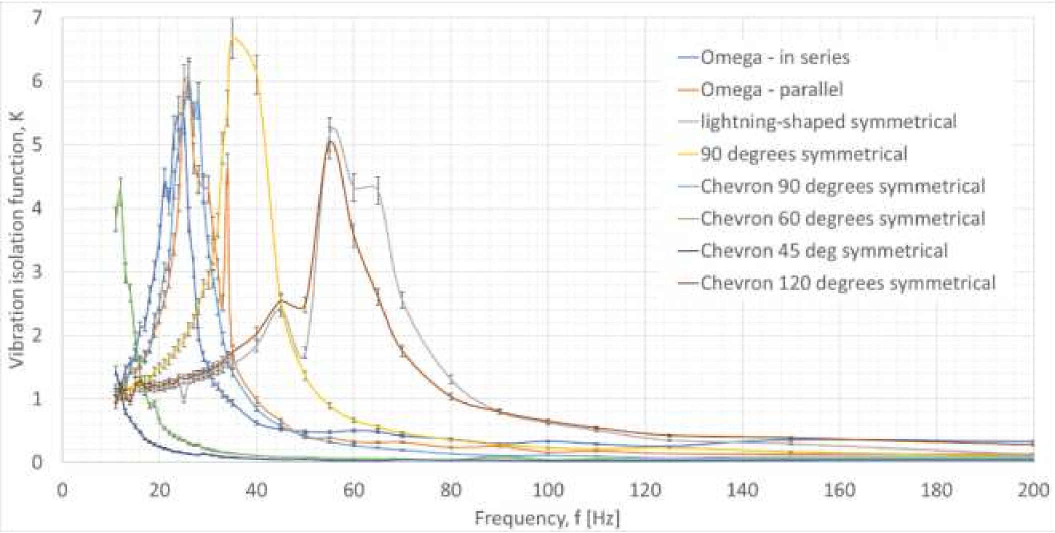

Fig. 2. Experimentally obtained vibration isolation functions of eight considered specimens with comparable wall thicknesses and different internal geometries.

Repeated measurements revealed changes in the resonance peak magnitude and, for selected geometries, shifts in resonance frequency. However, no uniform monotonic change in resonance frequency was observed for all specimens. The results demonstrate that the cross-sectional architecture of additively manufactured structures can be used to tune their low-frequency vibration-isolation behaviour. The changes observed during repeated loading also indicate the need for further investigation of fatigue resistance, structural durability, and the influence of manufacturing parameters.

Acknowledgments
This work was co-financed by the Polish Ministry of Science and Higher Education as a research project No. 0612/SBAD/3640 and No. 0612/SBAD/3656. The experimental tests were carried out at the Poznan University of Technology in Institute of Applied Mechanics.

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DOUBLE-BEAM PIEZOELECTRIC NON-LINEAR ENERGY HARVESTER SIMULATIONS

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ABSTRACT

The paper presents analytical and numerical models of a nonlinear double-beam energy harvester employing a piezoelectric generator.
The influence of the system's structural parameters on its dynamic characteristics under harmonic excitation is investigated. Dimensionless coefficients are introduced to describe the mechanical parameters of the energy harvesting system.

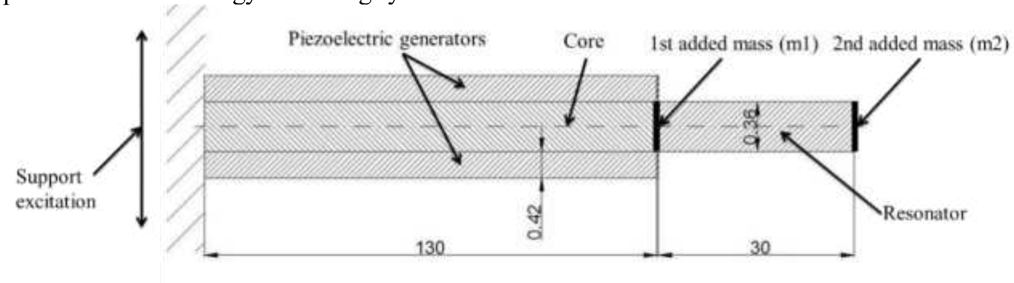

Figure 1. Scheme of the hybrid double-beam energy harvester

An analytical model is developed based on Timoshenko beam theory, accounting for nonlinear deformations in continuous and hybrid discrete–continuous systems. The proposed model is also examined using finite element method (FEM) software, with geometric nonlinearities of the hybrid structure taken into account.
Based on the characteristics describing the influence of dimensionless geometric parameters on the harvested voltage, desired ranges of the coefficients are identified, corresponding to structural features that are mechanically favorable from an engineering perspective.

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EXPERIMENTAL INVESTIGATION OF VIBROACOUSTIC RESPONSE IN A SLIDING SYSTEM WITH FRICTIONAL CONTACT

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ABSTRACT

This work presents an experimental test rig developed for investigating the vibroacoustic response of a mechanical system with sliding frictional contact. The study focuses on the relationship between imposed motion, slider dynamics, vibration response, and acoustic signals generated during frictional interaction. Frictional contact may introduce strong nonlinear effects, including stick-slip motion, friction-induced vibrations, irregular force transmission, and frictional noise. In real engineering systems, the dynamic response of a sliding contact is influenced by local contact conditions, surface roughness, assembly imperfections, loading history, and compliance in the excitation path. Therefore, dedicated experimental setups are required to observe these effects under controlled laboratory conditions and to provide data for model validation and diagnostic analysis.

The investigated system consists of a slider moving relative to a guide and a drive unit generating the relative motion by means of a belt drive controlled by a stepper motor. The drive position is measured using a rotary encoder, while the slider displacement is measured independently by a linear encoder. An important feature of the test rig is the flexible beam coupling between the drive and the slider. The general layout of the experimental test rig is shown in Fig. 1. The introduction of compliance into the excitation path increases the sensitivity of the system to local changes in frictional contact conditions and facilitates the observation of dynamic effects associated with friction.

For mechanical interpretation, the test rig can be represented by a simplified one-dimensional model in which the slider is treated as an equivalent mass coupled with the belt drive through a flexible beam. The slider dynamics may be written in the general form

$$m_s \ddot{x}_s(t) = k_b\big(x_d(t) - x_s(t)\big) + c_b\big(\dot{x}_d(t) - \dot{x}_s(t)\big) - F_f \tag{1}$$

where $x_d(t)$ is the imposed drive displacement, $x_s(t)$ is the slider displacement, m_s is the equivalent slider mass, k_b and c_b denote the equivalent stiffness and damping of the flexible beam, and F_f represents the resultant nonlinear friction force acting between the slider and the guide. This simplified physical model relates the imposed motion, compliant excitation path, frictional interaction, and measured dynamic response.

The measurement system is based on an NI USB-6341 data acquisition card and enables synchronized acquisition of signals from two accelerometers mounted on opposite sides of the slider, a measurement microphone, the drive system, and position sensors. A custom-built charge amplifier is used for conditioning signals from piezoelectric accelerometers before acquisition. All measurement channels are synchronized using a common reference clock, which enables consistent analysis of displacement, vibration, and acoustic signals recorded during sliding motion.

The proposed configuration enables investigation of friction-induced dynamic phenomena in a sliding mechanical system. Encoder signals describe the macroscopic motion, accelerometers provide information about the vibration response, and the microphone signal allows acoustic effects generated by frictional contact to be observed. The multisensor data may support identification of contact stiffness variations, stick-slip behaviour, friction-induced vibrations, and frictional noise generation. The obtained measurements can also be used for validation of frictional contact models and development of vibroacoustic diagnostic methods. The subject of the work is consistent with experimental techniques in sound and vibration engineering, signal processing and analysis, modelling and identification of dynamical systems, and nonlinear vibration phenomena [1–3].

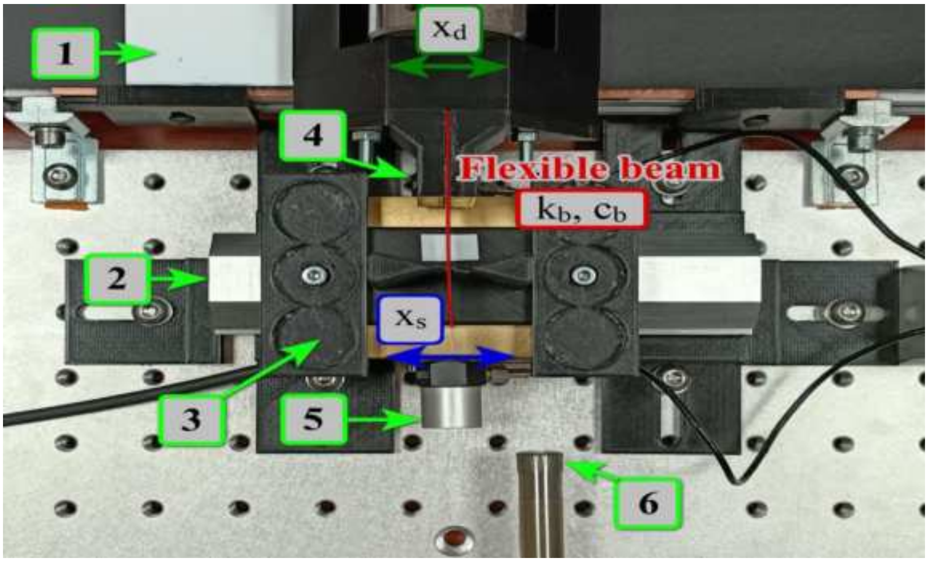

Fig. 1. Experimental test rig and mechanical interpretation of the sliding system with frictional contact: 1 – belt drive, 2 – guide, 3 – slider with displacement x_s, 4–5 – accelerometers, 6 – measurement microphone; x_d – drive displacement, k_b, c_b – equivalent stiffness and damping of the flexible beam.

Acknowledgments
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MAGNETIC PENDULUM UNDER EXTERNAL EXCITATION WITH STATE-DEPENDENT PHASE CONTROL

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ABSTRACT

This work investigates the dynamics of a strongly nonlinear magnetic pendulum subjected to external magnetic excitation with a state-dependent phase. Magnetic pendulums are known to exhibit a wide range of nonlinear responses, including periodic, multi-periodic, and chaotic oscillations [1], while state-dependent control of forcing parameters in non-autonomous oscillators may generate rich hierarchies of regular and irregular regimes [2]. In contrast to the classical formulation, where the excitation phase is constant, the phase of the coil current is here adjusted proportionally to the instantaneous angular position of the pendulum. This modification introduces a feedback-type phase modulation mechanism and fundamentally changes the timing of energy transfer between the electromagnetic actuator and the mechanical oscillator.

A mathematical model of the system is developed on the basis of an experimentally identified magnetic torque characteristic and includes gravity, linear elastic restoring torque, viscous damping, and nonlinear friction of Stribeck type. The model is validated against laboratory measurements for the classical constant-phase case, showing good qualitative agreement in phase trajectories, Poincaré sections, and bifurcation structure. On this basis, the influence of the phase-control coefficient on the system response is analyzed numerically.

The obtained results reveal a rich organization of periodic and chaotic regimes, broad families of coexisting attractors, and a significant reorganization of the bifurcation structure with respect to the classical magnetic pendulum. A particularly important result is the existence of a robust intrawell period-1 branch that persists over a wide range of excitation amplitudes despite substantial changes in the magnetic forcing level. Energy balance analysis shows that this response is sustained by localized bursts of magnetic power input concentrated near the lower equilibrium position, which compensate dissipation on a cycle-by-cycle basis.

For higher excitation amplitudes, the system exhibits multistability and intricate basin structures. Basin of attraction analysis reveals the coexistence of multiple periodic solutions and fractal basin boundaries, quantified using both box-counting dimension and boundary basin entropy. The study demonstrates that state-dependent phase control can be used as an efficient mechanism for attractor selection and regulation of energy transfer in nonlinear magneto-mechanical oscillators.

Acknowledgments

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SURROGATE-BASED MODAL SENSITIVITY ANALYSIS OF A FRAME WITH A FRACTIONAL KELVIN DAMPER

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ABSTRACT

Sensitivity analysis is one of the most important tools for assessing the influence of model parameters on the structural response. Its results can be used to identify parameters that are important from the design point of view, support optimization processes, assess the influence of input uncertainty, and facilitate model parameter identification. Therefore, sensitivity analysis can be regarded as one of the subsequent stages of the modern structural design process.

Sensitivity analysis becomes particularly important in dynamic problems involving structures with viscoelastic elements, which are widely used for vibration reduction. In the present study, the behavior of these elements is described using the fractional Kelvin model. The main objective of the study is to demonstrate that the sensitivity analysis of such systems with respect to changes in their design parameters can be effectively supported by machine learning methods, which are increasingly used in various fields of mechanics.

As a reference, analytical sensitivity measures were determined using the direct differentiation method (DDM). Both first- and second-order sensitivities were calculated [1]. The starting point is the eigenvalue problem for systems with viscoelastic elements formulated in the Laplace domain as:

$$\left(s^2\mathbf{M} + s\mathbf{C} + \mathbf{K} + \mathbf{G}(s)\right)\tilde{\mathbf{q}}(s) = \mathbf{0} \tag{1}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{C}$ is the damping matrix associated with structural damping, $\mathbf{K}$ is the stiffness matrix, s is the Laplace variable, $\tilde{\mathbf{q}}(s)$ denotes the Laplace transform of the displacement vector, and $\mathbf{G}(s)$ is the kernel-function matrix in the Laplace domain, whose form depends on the type of structure and the adopted rheological model. The nonlinear eigenvalue problem (1) is solved using the continuation method [2]. Its solution consists of complex eigenvalues and the corresponding eigenvectors. Subsequently, sensitivities of the complex eigenvalues are determined and used to evaluate the sensitivities of selected dynamic characteristics. In the present study, the sensitivities of natural frequencies are analyzed.

To compare the influence of parameters with different units and scales, the following dimensionless sensitivity measure is used:

$$S_p = \frac{p}{\omega}\frac{\partial \omega}{\partial p} \tag{2}$$

where p denotes the selected design parameter and ω is the natural frequency.

As a numerical example, a two-story frame equipped with a viscoelastic damper described by the fractional Kelvin model is considered (Fig. 1). For the analyzed system, a surrogate model based on Gaussian Process Regression (GPR) was constructed [3]. First- and second-order derivatives of the natural frequencies were then determined from the surrogate model using finite differences. This

approach enables repeated sensitivity analyses to be performed at different local points in the parameter space without the need to solve the full nonlinear eigenvalue problem each time.
The influence of design parameters on the structural response was also investigated using machine-learning model interpretation methods. For this purpose, Permutation Feature Importance (PFI) and Shapley Additive Explanations (SHAP) were employed [4]. The resulting parameter-importance measures were used to establish parameter rankings, which were subsequently compared with the rankings obtained from the dimensionless sensitivity coefficients determined from the analytical derivatives according to Eq. (2).
The obtained results showed very good agreement between the sensitivities determined from the surrogate model and those obtained analytically. The PFI and SHAP analyses also provided a consistent qualitative assessment of parameter importance. These results indicate that machine learning methods can effectively complement classical sensitivity analysis of structures with viscoelastic elements, particularly in problems requiring repeated computations, such as optimization, uncertainty analysis, and multi-variant structural design.

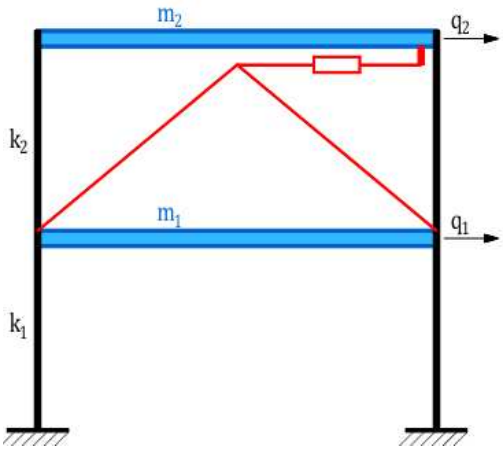

Fig. 1. Frame structure with a fractional Kelvin damper.

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IMPLEMENTATION OF USER-DEFINED ODE MODELS IN LIGGGHTS USING THE FIX FRAMEWORK FOR DEM SIMULATION OF A DUAL-CHAMBER ANTI-RESONANT VIBRATORY MILL WITH PARTICLE FRAGMENTATION

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ABSTRACT

The Discrete Element Method (DEM) has become one of the most widely used numerical techniques for modelling the behaviour of particulate materials. It is extensively applied to the simulation of particle transport, mixing, segregation, and comminution processes in both scientific research and industrial applications. Among the available DEM software packages, LIGGGHTS is widely recognized as one of the leading computational platforms and is available as the open-source LIGGGHTS-PUBLIC version and the extended LIGGGHTS Particulate Flow Module (LIGGGHTS-PFM).

LIGGGHTS-PUBLIC provides an open-source environment for simulating particle interactions using a wide range of contact models while allowing unrestricted access to the source code. Its principal advantages include transparency, flexibility, and the possibility of implementing user-defined physical models. However, the PUBLIC version offers limited functionality for advanced industrial applications and complex particulate flow simulations. LIGGGHTS-PFM extends the capabilities of the public version by providing additional tools dedicated to particulate flow modelling, comminution processes, and coupled multiphysics simulations. Both versions share the same DEM solver and software architecture, differing primarily in the range of available modules and advanced functionalities.

LIGGGHTS is implemented in C++ following the principles of object-oriented programming. Its software architecture consists of a hierarchy of interacting classes responsible for specific numerical tasks performed during the simulation. One of the key extensibility mechanisms is the Fix class, which enables additional operations to be executed during selected stages of the time integration procedure. By deriving new classes from the Fix framework, users can implement additional mathematical models, including systems of ordinary differential equations (ODEs), which are integrated alongside the standard DEM formulation. This modular approach allows the software to be extended without modifying the core solver while maintaining code flexibility and maintainability. The implementation of user-defined ODE systems is particularly valuable in the numerical modelling of vibratory mills. In such systems, the comminution process depends not only on particle interactions, effectively simulated using LIGGGHTS-PFM, but also on the dynamic coupling between the motion of the grinding chamber and the behaviour of the grinding media and processed material. Introducing dedicated ODE models through custom Fix classes makes it possible to account for the influence of the material load and its distribution on the machine dynamics, resulting in a more realistic representation of the operating conditions of vibratory mills and improving the predictive capability of DEM simulations.

The paper presents the results of numerical simulations of a dual-chamber anti-resonant vibratory mill, in which the grinding chambers follow a figure-eight trajectory. The simulations (Fig. 1) were performed using a two-way dynamic coupling between the grinding chambers and the particulate charge, enabling a realistic representation of the mill operating conditions. The analysis focuses on the key parameters governing the grinding process, with particular emphasis on the collision intensity between the grinding

media as well as between the grinding media and the processed material, which constitute the basis for evaluating comminution efficiency.

Furthermore, the particle fragmentation approach implemented in LIGGGHTS-PFM is presented. The fragmentation model is based on an energy criterion, whereby a particle fractures once the contact energy exceeds a prescribed critical threshold. The resulting fragments are generated stochastically while conserving mass, and their size distribution is determined according to the adopted particle size distribution model.

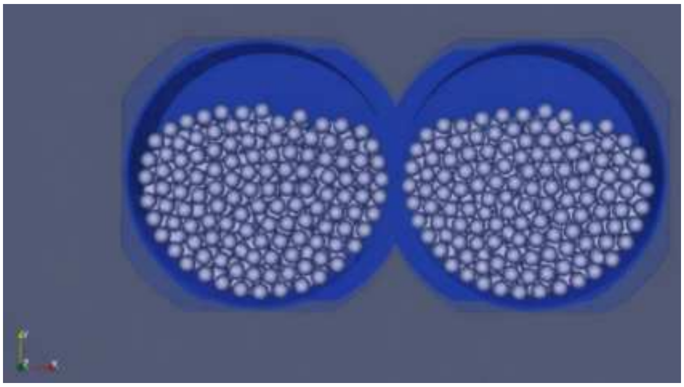

Fig. 1. Simulation of a dual-chamber anti-resonant vibratory mill.

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STUDY OF THE NEW FOOTBRIDGES AT THE ADAM MICKIEWICZ MUSEUM IN ŚMIEŁÓW IN TERMS OF DYNAMIC RESPONSE

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ABSTRACT

Footbridges may be susceptible to kinematic excitations generated by users (free walking, synchronized walking, running, jumping). When the frequency of the operational load aligns with the natural vibration frequency of the superstructure, resonance occurs, resulting in an increase in the deck's vibration accelerations. This reduces serviceability comfort.

This paper addresses the design challenges of footbridges located within the park of the Adam Mickiewicz Museum in Śmiełów. Three structural variants of a selected footbridge operating as a simply supported beam are analyzed (Fig. 1), namely: a) timber beam (three 18×28 cm beams, C30), b) steel beam (two HEA220 beams, S235), c) concrete slab (32 cm thick voided prestressed slab, C40/50). The primary objective is to evaluate the influence of the adopted solution on the dynamic response of the structure induced by pedestrians. The analysis also includes a comparison of construction and demolition costs, execution constraints, operational aspects, durability, and environmental impact. Based on this, an attempt is made to identify the optimal solution.

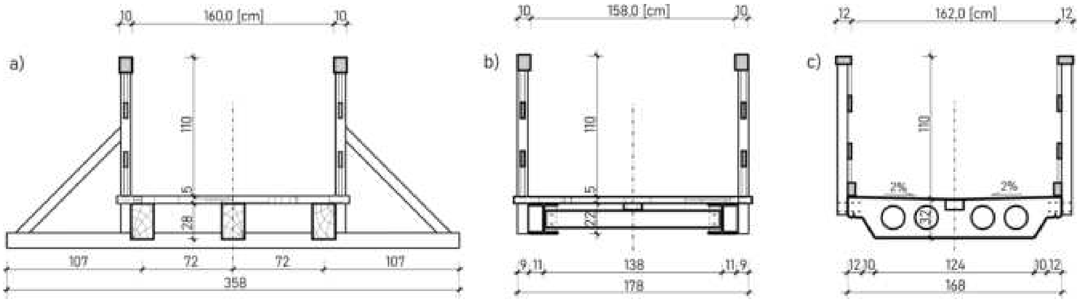

Fig. 1. Cross-sections of the analyzed footbridges (span length: 8.9 m)

Table 1. Risk classification of vertical resonance occurrence according to guidelines [2]

Risk of resonance occurrence	Natural vibration frequency [Hz]	Natural vibration frequency in the horizontal direction [Hz]
Maximum	1.7÷2.1	0.5÷1.1
Medium	1.0÷1.7 and 2.1÷2.6	0.3÷0.5 and 0.5÷1.1
Low	0÷1.0 and 2.6÷5.0	1.3÷2.5
Negligible	>5.0	0÷0.3 and >2.5

In European engineering practice, the standard [1] forms the basis for evaluating the dynamic response of structures, mandating the verification of vibration comfort criteria if the fundamental natural frequency of the structure does not exceed 5 Hz in the vertical direction and 2.5 Hz in the horizontal direction. The standard also defines the allowable deck acceleration values as 0.7 m/s^2 for the vertical direction and 0.2 m/s^2 for the horizontal direction. A significantly more detailed procedure is contained in the French

Sétra guidelines [2], which enable the assessment of resonance risk based on natural vibration frequencies (Table 1).
Analytical and computer methods were utilized in this study to compare the dynamic response of the structure with standards [1] and [2]. An evaluation was also conducted regarding the durability of the designed solutions, construction costs, execution constraints, and operational aspects. Such comparisons allow for the indication of an optimal variant that combines respect for the character of the park with ensuring the comfort, safety, and durability of the structure.
The dynamic characteristics of the analyzed footbridges were determined based on numerical analysis using the RSTAB software [3]. Models were created in the form of spatial frames. Natural vibration frequencies and vibration accelerations were calculated. Selected results of the analysis are summarized in Table 2.

Table 2. Calculated natural flexural vibration frequencies of the analyzed footbridges

	Structural variant of the footbridge according to Fig. 1		
	a)	b)	c)
Natural vibration frequency [Hz]	5.1	6.6	7.2

Table 3. Multi-criteria suitability assessment of the analyzed structural variants.

Criterion	Structural variant of the footbridge according to Fig. 1		
	a)	b)	c)
Comfort	Medium (highest susceptibility to vibrations)	Medium/high (better stiffness)	High (negligible excitability)
Indicative price [4]	Lowest (~3.5÷4.8 thousand PLN/m²)	Medium (~5.5÷7.5 thousand PLN/m²)	Highest (~8.5÷11 thousand PLN/m²)
Execution	Very easy (light equipment)	Medium difficulty (small mobile crane)	Very difficult (heavy equipment, logistics)
Maintenance	Costly, relatively frequent activities	Moderate (deck maintenance)	Negligible (high durability)
Demolition	Very easy and inexpensive	Easy (possibility of cutting the steel)	Very difficult and expensive
Environmental impact	Lowest (the most ecological material)	Medium (steel recycling)	Highest (carbon footprint, destruction of surroundings)

A multi-criteria suitability assessment of the analyzed structural variants is presented in Table 3. The cells corresponding to the best choice for each individual criterion are shaded. Within the scope of the considered criteria, the timber beam structure proves to be the optimal choice, although it is not the best option in terms of the user comfort (estimated natural frequency just above 5 Hz).

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NUMERICAL STUDY OF THE FEASIBILITY OF DETECTING LOCAL DELAMINATION IN A WIND TURBINE BLADE BASED ON MODE SHAPES

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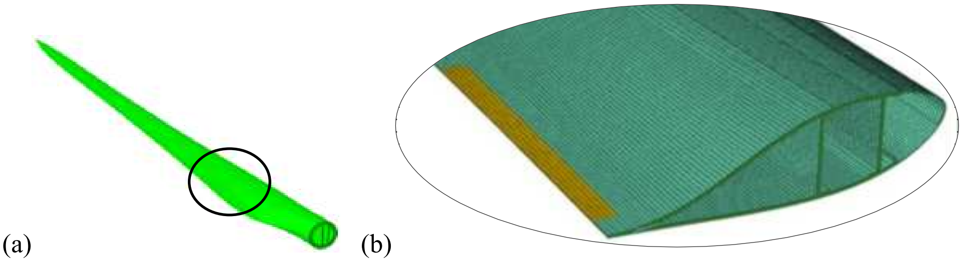
ABSTRACT

Contemporary wind turbine blades are fabricated from fibre-reinforced polymer composites, which provide high structural efficiency by combining high strength with relatively low mass. In cross-section, the blade forms a closed box, characterised by an aerodynamic profile at the leading edge and a flattened contour at the trailing edge. Along the span, this box section is typically stiffened by longitudinal shear webs. The entire outer shell is manufactured as a single laminate with locally tailored thicknesses and reinforcement patterns. The shear webs are subsequently bonded to the shell, after which the structure is closed and adhesively joined along the trailing edge. This manufacturing strategy minimises the extent of adhesive bonding, which is regarded as a region of reduced reliability due to production-related imperfections and potential material degradation [1]. Degradation of adhesive joints, together with leading-edge erosion and trailing-edge cracking, constitutes the primary structural mechanisms responsible for blade damage.

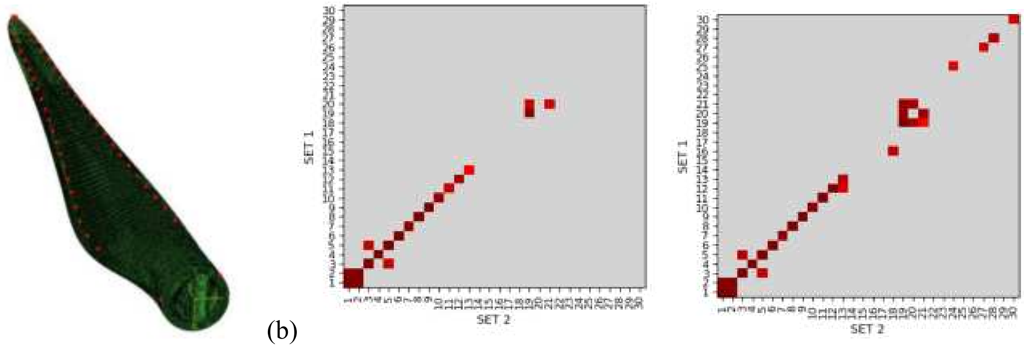
This study examines the feasibility of detecting local delamination at the trailing edge by means of modal shapes analysis. Two approaches are investigated. The first assesses the capability of identifying the occurrence of damage using the Modal Assurance Criterion (MAC) [2], which quantifies the similarity between two mode shapes—here, the mode shape of the undamaged structure and the corresponding mode shape of the structure with a local defect. The second approach evaluates the possibility of localising the delamination using the modal curvature method [3], in which local curvatures of the mode shapes obtained for the undamaged and damaged states of the structure are compared.

At this stage of the investigation, a numerical analysis was undertaken using the three-dimensional finite element blade model developed by Haselbach et al. [4, 5]. The model represents a full-scale 12.6-m wind turbine blade and is constructed from shell elements that explicitly capture all structural layers of the composite laminate, including the sandwich regions of the outer skin, the internal longitudinal shear webs, and the adhesive joints. Systematic delamination simulations were performed by removing the elements corresponding to the adhesive along the trailing edge (Fig. 1). Delaminations with lengths ranging from 0.2 m to 1.0 m, at successive positions along the blade were considered, each producing local perturbations in the mode shapes. The MAC analysis indicates that such perturbations become detectable for higher-order modes, see example result in Fig. 2 for delamination presented in Fig.1. The modal curvature method can also be effective for delamination localisation, provided that the spatial distribution of measurement points is selected appropriately with respect to the size of the delamination under consideration. The analysis performed here will serves as a reference for the planned experimental investigations concerning the feasibility of detecting local delamination using vibration-based methods.



(a) (b)

Fig. 1. (a) Geometry of the blade model; (b) insight to one cross-section with region of simulated delamination marked in yellow



(a) (b)

Fig. 2. (a) FEM model with 'measurement points' marked in red; (b) MAC values plot for delamination presented at Fig.1. The 0.9 threshold is set, purple color denotes '1' and lighter red denote values closer to 0.9. Left graph presents results for the trailing edge while the right one for the leading edge.

Acknowledgments

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GEOMETRICALLY NONLINEAR HENCKY BEAM WITH A TIP-MOUNTED SHEAR-THICKENING FLUID DAMPER UNDER HARMONIC BASE EXCITATION

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ABSTRACT

A geometrically nonlinear beam equipped with a tip-mounted shear-thickening fluid (STF) damper is investigated under harmonic transverse base excitation. To accurately capture large deflections, the beam is modeled using Hencky's discretization [1], which replaces the continuous structure with a chain of rigid links and rotational springs. The governing equations are derived using Lagrange's formulation in a nondimensional form and solved numerically via time integration while sweeping the excitation frequency. The study compares two right-end boundary conditions: a completely free tip and a horizontally constrained tip (allowed to move vertically but not longitudinally). We observe that horizontally constraining the tip stiffens the system, shifting the dominant resonance to higher frequencies and enhancing the nonlinear hardening behavior.

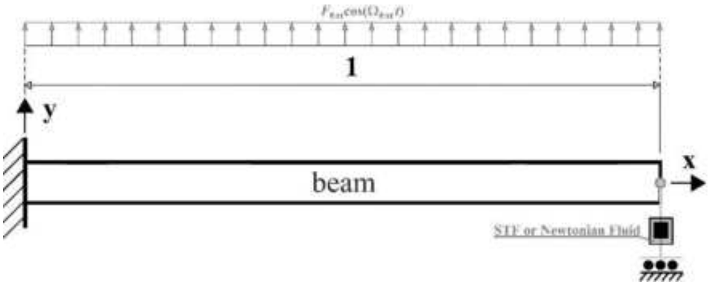

Fig. 1. Schematic of the free-end beam configuration with a tip-mounted damper.

The tip-mounted STF introduces a highly nonlinear damping force. Unlike traditional linear viscous dampers (LD), the effective viscosity of the STF increases drastically with the shear rate, making it a highly adaptive passive control mechanism. The nondimensional tip force generated by the STF is expressed as:

$$\hat{F}_{\mathrm{STF}}(t) = \alpha_{\mathrm{STF}}\, \dot{W}\ \hat{\eta}\ (\frac{\dot{W}}{\hat{h}}). \quad (1)$$

where α_{STF} is the nondimensional strength index, $\dot{w}$ is the tip velocity, $\hat{\eta}$ is the effective viscosity, and $\hat{h}$ is the normalized fluid gap height [2]. Previous studies have established the viability of STF dampers on purely linear beams driven by piezoelectric forces [3]. In contrast, our present work extends this foundation by explicitly comparing standard linear viscous dampers against nonlinear STF dampers in a fully nonlinear, large-deflection regime driven by base excitation.

Simulation results indicate that increasing the STF action significantly suppresses the main resonance peak, shifts it toward lower frequencies, and reduces the overall displacement across a broad frequency band. The response of an LD-supported beam (Fig. 2) shows a smooth, gradual attenuation of the primary resonance.

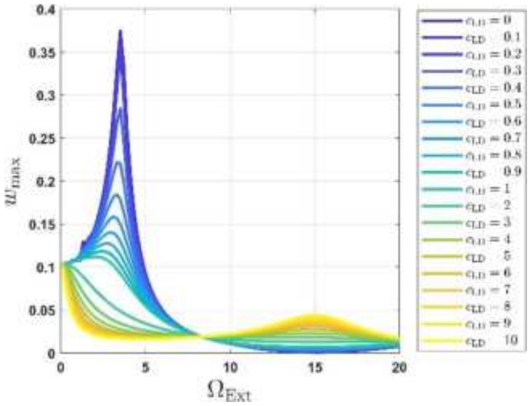

Fig. 2. Frequency response for the beam with varying linear damping (C_{LD}).

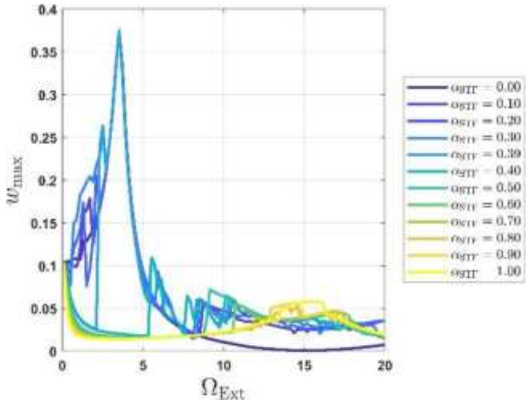

Fig. 3. Frequency response for the beam with varying α_{STF} at a fixed gap height.

By comparison, the STF-supported system (Fig. 3) provides substantially stronger peak suppression at similar operational configurations. However, for moderate STF action, the response may exhibit nonsmooth characteristics and separated response branches over limited frequency ranges. This multi-valued response indicates that more than one steady vibration level can exist for the same forcing frequency. As the STF action is further strengthened, either by increasing the damping index or narrowing the fluid gap, these post-resonance irregularities smooth out, leaving only a minor high-frequency hump. Ultimately, the results highlight the normalized fluid gap as a critical design parameter, as even minor geometric adjustments can drastically alter the damping efficiency. These findings demonstrate that STF tip dampers act as highly effective, self-adaptive passive devices for mitigating resonance-driven vibrations in slender composite structures.

Acknowledgments
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EBI - INDICATOR OF THE SOLUTION QUALITY OF A MECHANICAL DYNAMIC SYSTEM

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ABSTRACT

This paper discusses the EBI measure defined for numerical error assessment in dynamic system solutions [1]. The proposed definition of the measure is based on Newton's equation

$$m\ddot{x} = F_{pot} + F_{kin} + F_{vis} + F_{ext}, \tag{1}$$

and the law of conservation of energy applied to the flow of energy in a dynamic system interacting with its environment, containing potential [pot], kinematic [kin], viscous [vis], and external [ext] terms:

$$P_{pot} + P_{kin} + P_{vis} + P_{ext} = 0. \tag{2}$$

The procedure for calculating the EBI value begins with the appropriate calculation of the power of individual system terms, on the basis of which the energies accumulated by them are determined. For the physical system whose solutions satisfy the law of conservation of energy, the trend lines of the energies accumulated by the terms [pot] and [kin] have zero slope. Their energies oscillate without accumulation. The trend lines of the remaining terms have non-zero slopes, the sign of which depends on am increase (term [ext]) or a decrease (term [vis]) in the accumulated energy (Fig.1a).

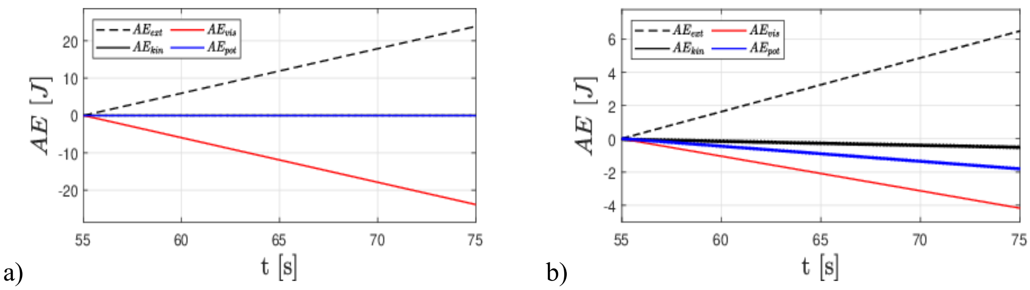

Fig. 1. Accumulation of flowing energy (AE) by individual system terms. a) system without significant solution errors, b) system with errors.

When the system solutions are disrupted by significant errors, the slopes of the trend lines of the energy accumulated by the terms [pot] and [kin] are non-zero (Fig. 1b). This effect is used in the definition of the EBI measure.

The paper presents the EBI values calculated for the numerical results of various dynamic systems obtained by different methods with forced inaccuracy. The EBI values are compared with the solution error results obtained by well-established methods. The comparison shows when errors in dynamic system solutions are significant.

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DISTRIBUTION OF ENERGY DISSIPATION ALONG A COUPLED OSCILLATOR CHAIN

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ABSTRACT

We study a chain of coupled oscillators with a view to using them as a tunable energy damper. Each oscillator is elastically coupled to its neighbours and includes viscous damping. A periodic driving force is attached to one end of the chain. By changing the spring constants of the couplings, the mass of the oscillators, and the damping constant, one can change the amount of energy dissipated by each element of the chain. We start by considering simple configurations of the possible mass distribution, see Fig. 1. In these cases the profile of the masses is linearly changing across the chain, changing the slope, but keeping the central mass fixed. This creates a particular profile of dissipation, with each mass dissipating differently. We note that there is no simple relationship between the dissipation and the mass profile, but by tuning the profiles one can target dissipation to particular sites in the chain.

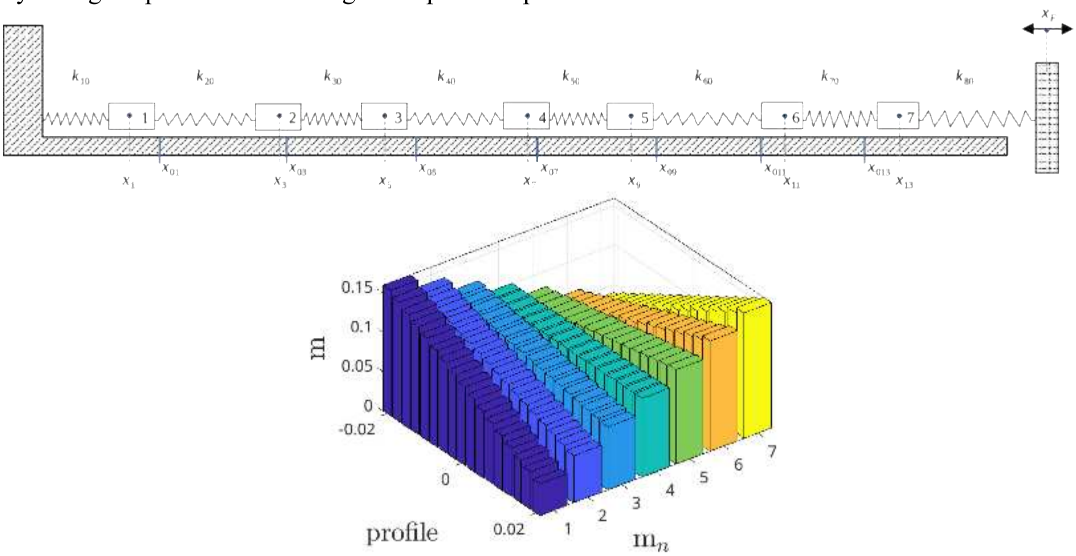

Fig. 1. Top panel shows a schematic of the set-up for a chain of seven oscillators. The bottom panel shows a set of possible mass profiles.

In addition to looking at the dissipation, we can decompose the energy into all of its different components, such as elastic and kinetic, for each oscillator individually [1]. We also consider how dissipation is altered by varying the full set of parameters, and by modifying the elastic terms, replacing them with non-linear forces. By tuning to situations where there are different dissipation profiles in the chain, we can create a broadband response in the system as a whole. In this case energy will be dissipated for a wide range of driving frequencies.

Acknowledgments
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ATTENUATION OF LONGITUDINAL ELASTIC WAVES IN A ROD BY AN AUTOPARAMETRIC PENDULUM ABSORBER – A MULTIPLE-SCALES ANALYSIS

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ABSTRACT

This study investigates the attenuation of longitudinal elastic waves in a rod by an attached autoparametric pendulum absorber. The coupled rod–pendulum model is derived using one-dimensional elastic-wave theory. The governing equations, which are geometrically nonlinear, are solved near the principal parametric resonance using a multiple-scale method, which allows for qualitative analysis. The asymptotic results are compared with the numerical results. The obtained results show that suitable tuning promotes energy transfer from the propagating wave to the pendulum and reduces the transmitted-wave amplitude. The effects of detuning, coupling, damping, and absorber parameters are examined. The proposed device provides a passive method for controlling wave propagation in slender elastic structures.

Mathematical model

The rod–pendulum system is modelled using one-dimensional elastic-wave theory. In the cross-section of the rod containing the point of attachment of the pendulum, the incident wave undergoes partial transmission and partial reflection, and the longitudinal motion of the rod excites the pendulum. The equations of motion take into account the conditions of compatibility of displacements at that cross-section and the conditions of equilibrium of an infinitesimal part of the rod surrounding the cross-section.

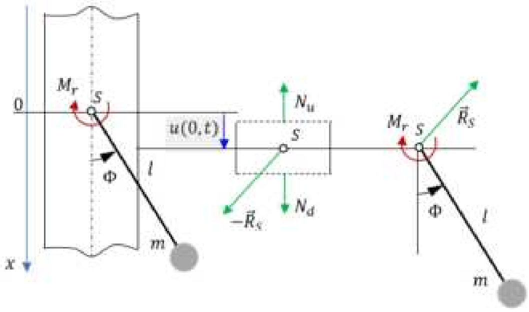

Fig. 1. Schematic of the elastic rod with an attached pendulum absorber.

After dimensionless variables are introduced, the suspension-point equation is

$$-a\,\gamma\,p\cos(p\tau)+\gamma\mu'(\tau)+\mu''(\tau)-\varphi''(\tau)\sin(\varphi(\tau))+\varphi'(\tau)^2(-\cos(\varphi(\tau)))=0 \tag{1}$$

The corresponding equation of motion of the pendulum is

$$c_d\,\varphi'(\tau)-\mu''(\tau)\sin(\varphi(\tau))+\varphi''(\tau)+\sin(\varphi(\tau))=0 \tag{2}$$

where $\mu(\tau)$ denotes the dimensionless displacement of the suspension point, $\varphi(\tau)$ is the pendulum angle, and a, γ, p, and c_d are dimensionless parameters defined by the excitation, damping, geometry, material properties, and mass distribution of the system.

Multiple-scales analysis and results

Equations (1) and (2) are analysed using the method of multiple scales (MMS). To describe the principal parametric resonance, the excitation-frequency parameter is written as $p = 2 + \sigma$, where σ denotes the detuning parameter. Higher-order solvability conditions lead to modulation equations for the slowly varying amplitude and phase of the pendulum. The asymptotic predictions were compared with direct numerical solutions of the original equations, achieving high agreement. Results, representative of the scope and purpose of the paper, are depicted in Fig. 2, which shows that the pendulum's oscillation amplitude, initially relatively small, increases significantly and achieves the steady state. This means transferring energy from the longitudinal wave in the rod to the absorber. Under suitable tuning conditions, this transfer leads to a clear reduction in the transmitted-wave amplitude.

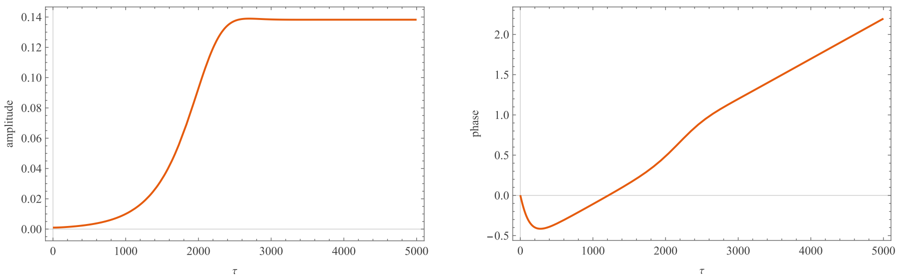

Fig. 2. Time histories of the amplitude and phase of the pendulum absorber for $a = 0.01$, $\gamma = 0.995$, $c_d = 0.004$, $\sigma = 0.001$.

The effects of excitation frequency, absorber parameters, coupling, detuning, and damping are examined. The results show that attenuation is sensitive to tuning and is effective within a limited parameter range. The pendulum absorber therefore offers a promising passive method for reducing longitudinal wave transmission in slender rods.

Acknowledgments

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STIFFNESS, DAMPING AND INERTIA FORCES COEFFICIENTS OF THE OIL FILM OF TILTING 5-PAD JOURNAL BEARINGS

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ABSTRACT

The dynamic characteristics of journal bearings are basically defined by the stiffness and damping coefficients of the oil film and they are the basis of stability determination in rotor dynamics [1,2]. However, more exact characteristics can be obtained by including the oil film inertia forces coefficients. Tilting 5-pad journal bearings are the bearings for high-speed operation and the determination of their stability should include the inertia forces coefficients of oil film. All above-given coefficients assure more exact calculation of stability of multilobe and titling-pad journal bearings. Exemplary results of calculated inertia forces coefficients shows Figure 1; rotational speed of journal equal to50 krpm (1 krpm equal to 1000 rpm).

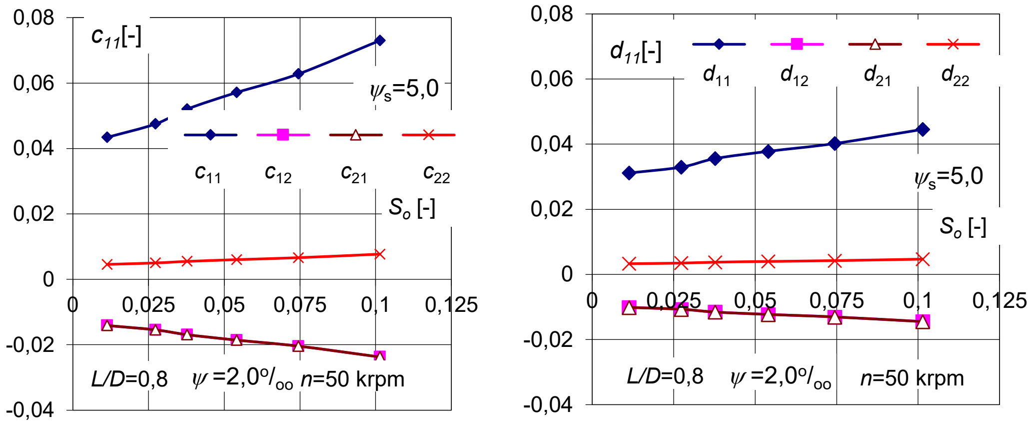

Fig. 1. Inertia forces coefficients c_{ik} and d_{ik} of oil film versus Sommerfeld number S_o in case of tilting 5-pad journal bearing (bearing relative: length L/D, clearance ψ, pad clearance ψ_s

The paper presents the results of the calculation of stiffness, damping and inertia coefficients of the tilting 5-pad journal bearing. The inertia forces of adiabatic oil film of bearing with finite length were considered. The Reynolds and energy equations comprise the correction coefficients of turbulence and were solved by means of numerical algorithm on the assumption of static equilibrium position of the journal. The coefficients were determined be means of perturbation method.

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DAMPED VIBRATIONS OF A Γ-TYPE FRAME AT SELECTED TYPES OF CONSERVATIVE LOADING

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ABSTRACT

Dynamic analysis of slender structural systems is fundamental to ensuring their safety and operational reliability. Due to their geometry, these structures are particularly susceptible to loss of stability and resonance under dynamic loads, making precise modelling of their vibration response crucial.
This paper presents theoretical considerations and numerical studies of the transverse-longitudinal damped vibrations of a Γ-frame. Particular attention was paid to damping modelling, taking into account its three fundamental types: structural, internal, and external damping [1, 2]. This approach allows for a more accurate representation of the actual operating conditions of the structure.
The frame is loaded with a generalized conservative load with a force directed towards the positive pole [3]. The load-carrying system consists of circular heads: a receiving head and a loading head (as segments of rolling bearings) [4]. A key aspect of the model is the ability to manipulate the diameters of these heads. By selecting them, the model allows for the simulation of various conservative load cases, such as a follower force directed to the pole, a force directed to the pole, or a classical Euler's load.
The vibration problem was formulated based on Hamilton's principle, after defining the potential and kinematic energy of the systems under consideration. Using solutions to the differential equations of motion, a transcendental equation for the damped vibration frequency was derived. As a result, the conducted research allows for a detailed comparative analysis that assesses the impact of individual damping types on the dynamic properties of the tested frames. This analysis is conducted in the context of various conservative loading scenarios, giving the work a broad scientific scope.

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INFLUENCE OF BEAM FLEXIBILITY MODELING METHODS ON THE SIMULATION OF IMPACT-INDUCED VIBRATIONS

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ABSTRACT

In mechanical engineering, discretization methods are most commonly used to model the flexibility of machine parts. These methods differ in terms of the complexity of their mathematical formulations, the accuracy with which they represent reality, and their computational efficiency. This paper presents the application of the Polish Rigid Finite Element Method (RFEM) to simulate of a flexible beam striking an obstacle. Two approaches to this method are compared: the classical one (RFEMc) [1] and modified one (RFEMm) [2].

Fig. 1 presents the beam under consideration – a physical discretized into rigid finite elements (rfe *i*) connected by spring-damping elements (sde *i*), in both classical (RFEMc) and modified (RFEMm) formulations. Using both models, a simulation is performed of dynamics of the pendulum falling from the initial position E_0 to the position E_1, where it strikes an spring-damping obstacle at point C, located at a distance (l_C) from the beam attachment point. The formalism of joint coordinates and homogeneous transformation matrices is applied to describe the beam model [3]. The vector of generalised coordinates consists of the following components

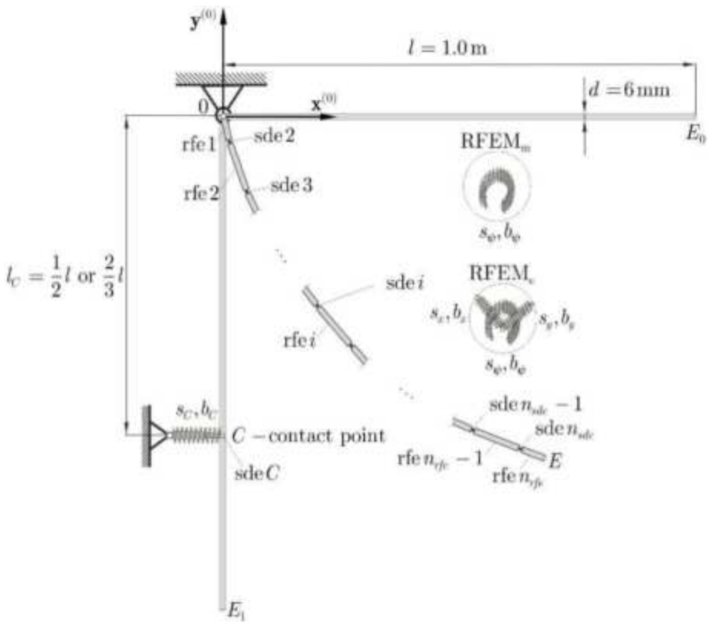

Fig. 1. Model of the flexible beam

$$\boldsymbol{q} = \begin{cases} \hat{\psi}^{(1)} & \text{– rigid beam,} \\ \left[\hat{\psi}^{(1)} \quad \hat{\psi}^{(2)} \quad \dots \quad \hat{\psi}^{(n_{rfe})}\right]^{\mathrm{T}} & \text{– RFEM}_{\mathrm{m}}, \\ \left[\hat{\psi}^{(1)} \quad \tilde{x}^{(2)} \quad \tilde{y}^{(2)} \quad \tilde{\psi}^{(2)} \quad \dots \quad \tilde{x}^{(n_{rfe})} \quad \tilde{y}^{(n_{rfe})} \quad \tilde{\psi}^{(n_{rfe})}\right]^{\mathrm{T}} & \text{– RFEM}_{\mathrm{c}}, \end{cases} \tag{1}$$

The Lagrange equations of the second kind are used to derive the dynamics equations. They can be written in a general matrix form in the following form

$$\mathbf{M}\ddot{\mathbf{q}} = \mathbf{h}, \qquad (2)$$

where: **M** is the mass matrix, **h** is the vector of the right side of the dynamics equations. The algorithms for generating **M** and **h** matrices are presented in [4].

Fig. 2. presents some exemplary results – specifically, the time courses of the coordinates of the beam tip *E* – for cases where the beam is treated as rigid and as flexible (with flexibility modeled using both classical and modified RFEM approaches), considering obstacles located at different l_C distances from the tip *0*.

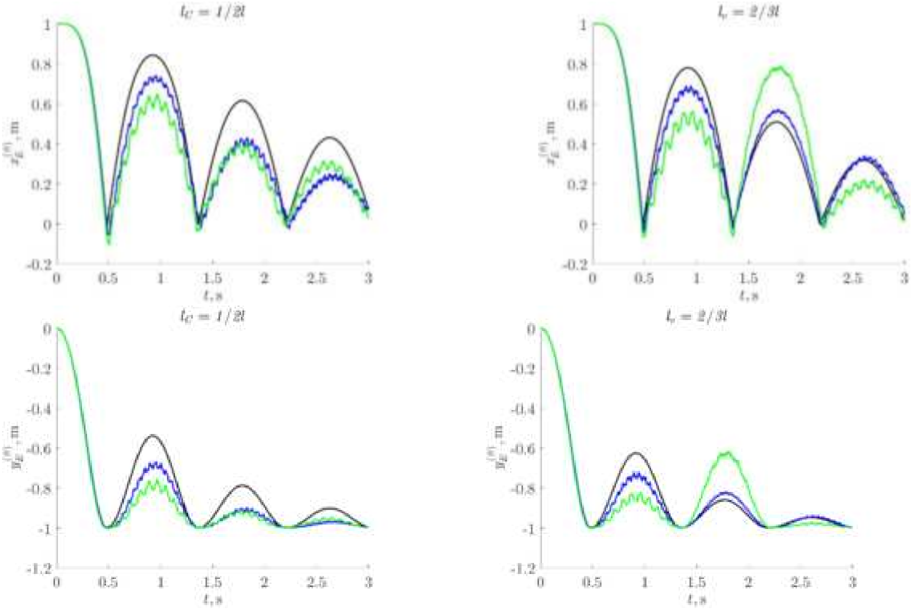

Fig. 2. Time courses of the coordinates of the beam tip *E*.

The models of the analyzed beam described in the abstract are used to conduct a series of simulations, based on which the influence of the applied methods for modeling beam flexibility on the following was evaluated:

- ➢ accuracy in representing reality,
- ➢ suitability for engineering calculations,
- ➢ numerical efficiency in simulation.

These issues will be discussed in greater detail during the conference.

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STOCHASTIC GAIT ANALYSIS WITH USE KINETIC PARAMETERS

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ABSTRACT

This paper presents a probabilistic gait analysis using standard measurement tools such as the BTS Smart motion capture system and dynamometric platforms. The study group consisted of several dozen young adults whose free gait was recorded along a several-meter-long measurement path. Key dynamic parameters typically used in gait analysis were analyzed: characteristic extreme values of the vertical component of the ground reaction force and the anteroposterior ground reaction, measured using dynamometric platforms. These parameters were stochastically analyzed depending on the subject's weight and average walking speed. A Gaussian probability distribution function $g(x)$ of the observed design parameter is assumed. The estimated dynamic parameters were determined using random analysis using three independent approaches: semi-analytical method (SAM), stochastic perturbation technique(SPT), and finally Monte-Carlo simulation (MCs).
The results obtained using all the presented methods were very similar and allowed for the effective estimation of the desired random parameters. The analysis itself provided valuable information regarding deviations from the desired normal distribution of gait parameters, treated as random variables in the analysis. The probabilistic approach allows for the identification of individuals who outlier from the population based on the analyzed parameters, making the target group more homogeneous. Random analysis allows for the easy identification of incorrect gait recordings (e.g., the subject pressing only part of the foot on the platform, stumbling, or slipping), which are easily missed during retrospective analysis. Furthermore, it is possible to identify individuals whose gait parameters deviate from the population for other reasons, which, given the correct recording, may indicate an undiagnosed musculoskeletal problem. Having this, probabilistic moments are calculated, i.e. the expectation, standard deviation and coefficient of variation [1]:

$$E(a) = \int_{-\infty}^{+\infty} \sum_{j=0}^{n} C_{ij} v^j g(x)dx,\ \sigma(a) = \left\{\int_{-\alpha}^{\alpha} \left(\sum_{j=0}^{n} C_{ij} v^j - E[a]\right)^2 g(x)dx\right\}^{\frac{1}{2}},$$
$$\alpha(a) = \left|\frac{\sigma(a)}{E(a)}\right|,\ \beta(a) = \frac{\mu_3(a)}{\sigma^3(a)},\ \kappa(a) = \frac{\mu_4(a)}{\sigma^4(a)}. \quad (1)$$

Vertical component of GRF in % of body weight is investigated. A set of eleven deterministic points was adopted – mass values and the corresponding percentages of the force acting on the one limb during walking. The mass values vary irregularly from 65.8 kg to 83.12 kg. The continuous response of the system was described by a third-degree fitting polynomial, the coefficients of which were calculated using the classical least squares method (LSM). The calculation results are presented in Figs. 1 and 2.

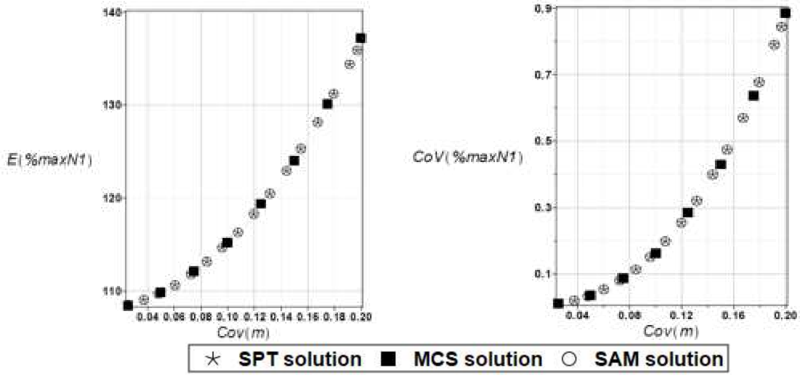

Fig. 1. Solution for right lower limb: expected value and coefficient of variation for the random distribution of human mass

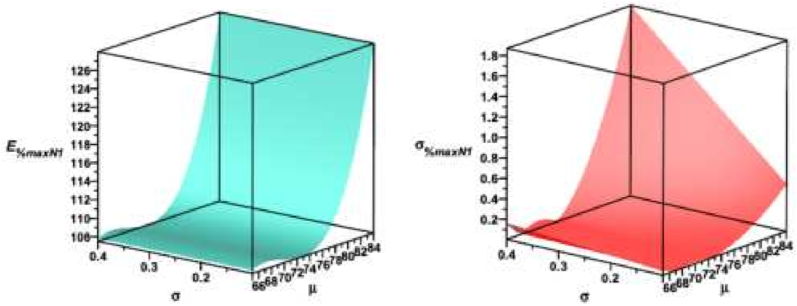

Fig. 2. Solution for right lower limb: expected value and standard deviation for the random distribution of human mass

The approach presented in this paper can be used to clean data, for example, before the augmentation procedure, which is very often used in preprocessing machine learning methods. It can be also observed individuals who have had a musculoskeletal disorder and have not declared it or have a problem with it that requires further diagnosis.

Acknowledgments
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VIBRATIONS AND CHAOTIC MOTION INDUCED BY ANISOTROPIC FRICTION IN THE HORIZONTAL PENDULUM ON A ROTATING DISK SYSTEM

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ABSTRACT

Anisotropic friction can act as a source of vibrations and chaotic behavior in mechanical systems. To investigate this phenomenon, a dimensionless model of the specific mechanical system is developed in which the tip of a horizontal pendulum slides on a rotating disk, with anisotropy introduced through the surface structure of the disk (see Fig. 1). The considered setup can serve as an archetypal system for studying a wide range of practical applications, including disk brake operation and clutch dynamics.

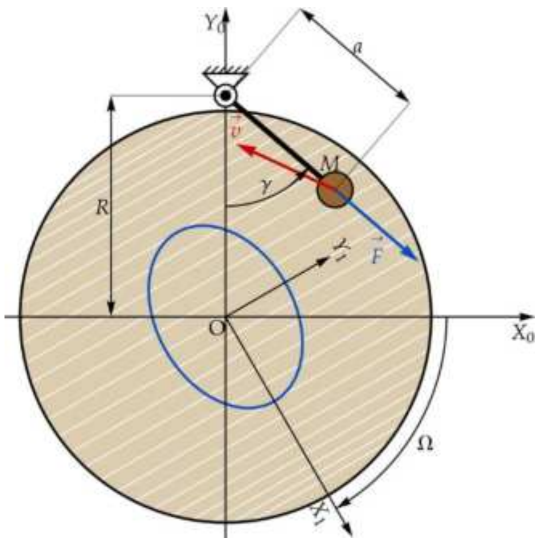

Fig. 1. Horizontal pendulum in frictional contact with a rotating disk.

The results show that when the friction force is not aligned with the direction of sliding, the system loses its static equilibrium and evolves toward limit cycle oscillations. With variation of system parameters, these oscillations undergo a sequence of period-doubling bifurcations, eventually leading to the formation of chaotic attractors. The emergence of chaos is robust within bounded regions of the parameter space, although it remains sensitive to additional sources of energy dissipation [1].

The mechanism responsible for the onset of periodic motion is explained using a reduced-order parametric model derived from the linearized counterpart of the system [2]. Both analytical and numerical predictions are validated experimentally using a dedicated laboratory setup.
Although anisotropic friction is capable of inducing chaotic dynamics, the required conditions—such as a high degree of anisotropy combined with low bearing friction—are not commonly encountered in typical engineering applications. Nevertheless, the findings highlight that surface anisotropy can be treated as a controllable design parameter. On one hand, it may be exploited to generate complex responses, for example in energy harvesting systems; on the other hand, it should be minimized in applications where stable and predictable performance is essential.

Acknowledgments
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INFLUENCE OF PRONY SERIES MODEL ORDER ON THE FREQUENCY DEPENDENT RESPONSE OF VISCOELASTIC DAMPERS

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ABSTRACT

Viscoelastic materials are widely used in vibration mitigation systems due to their ability to provide both stiffness and energy dissipation. In finite element analyses, their time-dependent behavior is commonly represented using Prony-series formulations calibrated from Dynamic Mechanical Analysis (DMA) data. While considerable attention has been devoted to improving the accuracy of Prony-series fitting procedures, the influence of model complexity on structure-level dynamic response remains less well quantified.

$$G(t) = G_\infty + \sum_{\mathrm{i}=1}^{\mathrm{N}} G_i \mathrm{e}^{\left(-\frac{t}{\tau_i}\right)} \tag{1}$$

In this Eq. (1), $G(t)$ represents the time-dependent shear modulus of the material. The parameter G_∞ denotes the long-term shear modulus as time approaches infinity, representing the purely elastic response of the material. The summation terms capture the transient viscoelastic behavior, where G_i represents the shear relaxation modulus of the i-th Maxwell element, and τ_i is its corresponding relaxation time. The ratio t/τ_i indicates the rate of stress relaxation over time t, with N representing the total number of terms in the series required to accurately capture the material's viscoelastic behavior.
This study investigates whether increasing the complexity of Prony-series viscoelastic models (i.e. increasing the number of terms N) leads to meaningful improvements in the dynamic response quantities relevant to vibration mitigation. The analysis is performed using two commercially available viscoelastic materials, Getzner Sylodamp SP500 and SP1000. Experimental DMA measurements of storage and loss moduli within the frequency range of 0.1–30 Hz are used to derive viscoelastic material models with varying Prony-series orders. The calibrated models are implemented in Abaqus and applied to a shear-type damper consisting of viscoelastic layers bonded between steel plates.

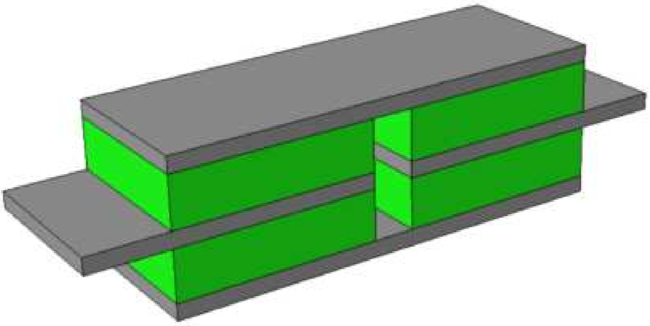

Fig. 1. Numerical model of the viscoelastic damper.

Harmonic analyses are performed over a range of excitation frequencies corresponding to the experimental characterization data and for varying displacement amplitudes. The influence of Prony-series order is evaluated through dynamic stiffness, hysteretic energy dissipation, and equivalent damping characteristics.

The obtained results indicate that increasing the number of Prony-series terms does not necessarily lead to a proportional improvement in the structural dynamic response. While higher order representations may improve agreement with experimental DMA data in certain frequency regions, this improvement is not consistently reflected in every structural dynamic response metrics.

Acknowledgments

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DETERMINATION OF DYNAMIC CHARACTERISTICS OF FRAMES WITH KELVIN DAMPERS AND UNCERTAIN PARAMETERS USING INTERVAL ANALYSIS

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ABSTRACT

In the classical design process, the values of design parameters are usually treated as deterministic quantities. In real structures, however, these parameters are subject to uncertainty, which may result, for example, from material heterogeneity, construction imperfections, the influence of operating conditions, or inaccuracies in parameter identification. Neglecting these uncertainties may lead to an inaccurate prediction of the structural response. This study considers the eigenvalue problem for structures equipped with viscoelastic dampers, in which the parameters are uncertain but their uncertainty intervals are known. A quadratic eigenvalue problem is analysed, whose solution consists of the eigenvalues and the corresponding eigenvectors.

A shear frame equipped with dampers described by the Kelvin model is analysed (Fig. 1). Each damper consists of a spring with stiffness k_0 and a damping element with damping coefficient c_0, connected in parallel. After applying the Laplace transform, the eigenvalue problem of the analysed structure takes the following form:

$$(s^2\mathbf{M}+s\mathbf{C}+\mathbf{K})\mathbf{q} = \mathbf{0} \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are the mass, damping, and stiffness matrices, respectively. The damping and stiffness matrices are composed of contributions associated with the structure and those resulting from the Kelvin dampers. The solution of the quadratic eigenvalue problem (1) consists of complex eigenvalues and the corresponding complex eigenvectors, which can be used to determine further dynamic characteristics, such as natural frequencies and nondimensional damping ratios. In the analysis, the design parameters are assumed to be represented by interval numbers:

$$p_i^I \in \left[\underline{p_i}, \overline{p_i}\right], i = 1, \dots, n_p \tag{2}$$

where n_p denotes the number of parameters, while $\underline{p_i}$ and $\overline{p_i}$ are the lower and upper bounds of the i-th parameter, respectively. The uncertainty of the parameters is therefore described using interval analysis, originally introduced by Moore [1]. Consequently, the structural response, expressed in terms of dynamic characteristics, is also obtained in interval form [2-4]. However, the direct application of interval arithmetic usually leads to a significant overestimation of the resulting bounds. This is caused, among other factors, by the fact that the same physical quantity appears multiple times in the matrices of the eigenvalue problem (1), whereas classical interval arithmetic treats each occurrence as an independent interval quantity. The main objective is therefore to reduce this overestimation. For this purpose, the quadratic eigenvalue problem for interval parameters is rewritten as a generalized linear eigenvalue problem:

$$(\mathbf{A}^I - s^I\mathbf{B}^I)\mathbf{z}^I = \mathbf{0} \tag{3}$$

where

$$\mathbf{A}^I = \begin{bmatrix} -\mathbf{K}^I & \mathbf{0} \\ \mathbf{0} & \mathbf{M}^I \end{bmatrix}, \mathbf{B}^I = \begin{bmatrix} \mathbf{C}^I & \mathbf{M}^I \\ \mathbf{M}^I & \mathbf{0} \end{bmatrix}, \mathbf{z}^I = \begin{bmatrix} \mathbf{q}^I \\ s^I\mathbf{q}^I \end{bmatrix} \tag{4}$$

The interval matrices, eigenvalue, and eigenvector are decomposed into their central values and uncertain increments:

$$\mathbf{M}^I = \mathbf{M}^C + \Delta\mathbf{M}, \mathbf{C}^I = \mathbf{C}^C + \Delta\mathbf{C}, \mathbf{K}^I = \mathbf{K}^C + \Delta\mathbf{K}, s^I = s^C + \Delta s, \mathbf{q}^I = \mathbf{q}^C + \Delta\mathbf{q} \tag{5}$$

where the superscript c denotes quantities evaluated at the central values of the parameters. After substituting these expressions into the eigenvalue problem (3) and neglecting products containing at least two increments, a linear system with respect to the changes in the eigenvalue and eigenvector is obtained. In general form, it can be written as:

$$\mathbf{L}^C\,\Delta\tilde{\mathbf{z}} = \sum_{i=1}^{n_p} \mathbf{P}_i^c \Delta p_i \tag{6}$$

where $\mathbf{L}^C$ is the matrix evaluated for the central values of the parameters, whereas $\mathbf{P}_i^c \Delta p_i$ describes the contribution of the i-th interval parameter and $\Delta\tilde{\mathbf{z}}$ is the vector of unknowns. The results of the calculation will be compared with those obtained using the vertex method, adopted as a reference solution [3,4].

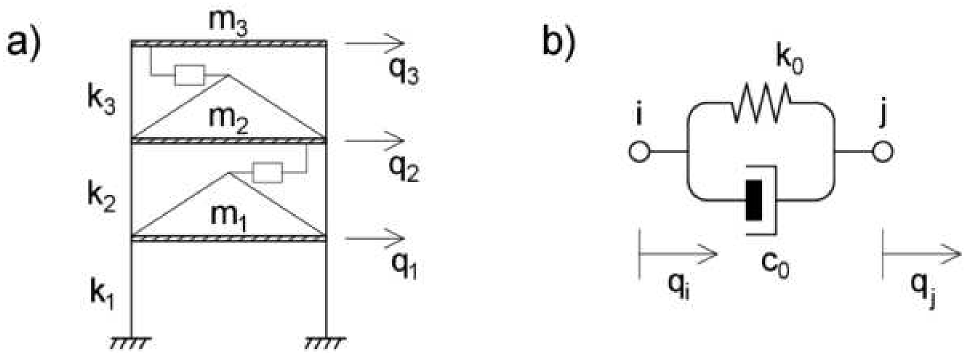

Fig. 1. a) Scheme of the analysed frame with dampers and b) the classical Kelvin rheological model.

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